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Capital budgeting with an efficient yield-based method: *the real rate of return technique*

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Abstract: We develop a yield-based capital budgeting method that solves the inconsistencies of the internal rate of return (IRR) and its alternatives with the shareholders' wealth maximization objective. We thus provide an efficient technique for managers who exhibit in practice a large preference for comparing the merits of projects with rates of return. This new method, called the real rate of return (RRR), is an improvement of the modified internal rate of return (MIRR) based on the Fisher equation. Simple and fitting with managers' needs and way of thinking, the RRR has all the qualities to be accepted in practice.

Keywords: Capital budgeting, modified internal rate of return, net present value, profitability index, Fisher equation.

JEL Classification Numbers: G31.

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1 Introduction

Recent capital budgeting surveys indicate that yield-based capital budgeting techniques are widely used in practice by managers (see among others: Arnold and Hatzopoulos 2000, Graham and Harvey 2001, Brounen et al. 2004, Meier and Tarhan 2007). In the United States, for example, the internal rate of return (IRR) is the managers' preferred method for project evaluation before the net present value (NPV)[1]. From an academic point of view, this finding is surprising because it is a clearly established fact that the IRR suffers from many drawbacks which could lead firms to bad decisions when capital is unlimited or rationed. On account of the absence of an efficient yield-based capital budgeting method, corporate finance textbooks recommend that managers should use, in place of the IRR, the NPV when the capital is unlimited and the profitability index (PI) when the capital is rationed[2] (see for instance, Fabozzi and Peterson 2003, Brealey et al. 2004, Ross et al. 2005)[3].

Although capital budgeting surveys do not indicate in which cases and how these methods are used (capital rationed or not, IRR used knowing its drawbacks or not), we can conclude that there exists a theory-practice gap. It is true that the NPV has been increasingly adopted by managers since a few years, but the use of the PI remains marginal, at around 10% depending on the country. In practice, firms' internal funds and debt capacity are not unlimited, and so firms cannot achieve all identified projects with a positive NPV. For example, survey results from Arnold and Hatzopoulos (2000) on some British firms indicate that 50% of them put budget ceilings on their production units (internal capital rationing).

[1] The IRR is the "first choice" capital budgeting technique in the survey of Meier and Tarhan (2007), and is the first "always or almost always used" technique according to the questionnaire of Graham and Harvey (2001). In Europe, the IRR is also popular but, according to the country, this method and the NPV are only at the second or third place after the payback period rule (simple payback period, not the discounted payback period).

[2] Notice that the choice of the optimal group of projects is not always easy to determine with the PI. This arises when capital is rationed over more than one period, or when other resources than capital (employees, production capacity, etc.) are also rationed, or even when projects are not independent for some other reasons. In such cases, the optimal group of projects is obtained with tedious trial and error or with dynamic programming.

[3] In this paper, we consider only now or never decisions (a rejected project is not realizable in the future) and ignore the existence of embedded real options.

Obviously, the PI is underused by managers who prefer the IRR and its drawbacks to assess projects profitability per invested euro.

This managers' preference for yield-based criteria has two explanations: habit and risk perception. The world of business and finance has always thought with rate-based measures: bank loans, financial investment, annual sales growth, etc. As a result, to summarize and compare the merits of a project, it is from a cognitive point of view more natural for managers to communicate with percentages (IRR) rather than amount of money (NPV) or with an index (PI). In addition, managers like to be able to compare the project's expected rate of return with the opportunity cost of capital because this gives them an assessment of the degree of risks they take by accepting the project. Thus, a risk averse manager is all the more likely to accept a project since its rate of return is important relative to the opportunity cost of capital required for this type of project. Furthermore, under uncertainty and from experience, managers prefer yield-based criteria because they limit the opportunities to invest in projects with large NPV but small returns, which might be oversized compared to the future growth of the market. For these reasons, it is very usual to see managers basing their investment decisions solely on yield-based methods even if there is no real limit on the capital available.

Despite this managers' preference, no rate of return is able to maximize shareholders' wealth under capital rationing. In other words, there is to our knowledge no alternative to the PI which would be the complementary criterion to the NPV. Among the many improvements of the IRR, the modified internal rate of return (MIRR) (Lin 1976, Mc Daniel et al. 1988) has many qualities explaining its introduction in finance textbooks and its adoption in some companies.

Unfortunately, when costs of capital differ between projects, the MIRR does not rank projects consistently with the PI and the NPV[4]. This drawback severely limits the application fields of the MIRR because, in practice, costs of capital are very likely to be different between

[4]Notice that this inconsistency with the NPV also arises when there is no scale difference between projects, situation for which the PI is consistent with the NPV.

projects. Indeed, the cost of capital is calculated from the relative weights of each component of the company's capital structure (shares of debt and equity), from the firm's market risk (or from division's market risk) and from the risk of the project. Thus, in a firm, costs of capital of evaluated projects are likely to be different because of the adjustment of the weighted average cost of capital (WACC) to the project's risk. In an investment bank, the probability that costs of capital differ between projects is all the more important since companies wishing to be funded have in principle different WACC.

In this paper, we develop a yield-based capital budgeting technique that solves the problem of disparity between projects costs of capital and also the other drawbacks of the IRR and its variants. Therefore, our method maximizes shareholders' wealth when capital is rationed and can be used in place of the PI as a complementary tool to the NPV.

The paper is organized as follows. First, we present the problem to be solved by surveying the literature about the drawbacks of the IRR and its variants. Second, we expose our method, give its economic interpretation and demonstrate its economic efficiency. Third, we show that when cash flows are uncertain, our method is also less biased than the MIRR. Finally, we conclude on the practical implementation of our capital budgeting technique.

2 Definition of the problem

Following the surveyed literature, we can say that an evaluation technique is efficient if it fulfils all the following conditions:

a. Give a unique solution and therefore a unique decision rule for every project;

b. Consider all cash flows;

c. Consider the timing of cash flows and the time value of money;

d. Consider the riskiness of cash flows;

e. Rank projects with different lifetime in order to ensure the maximization of shareholders' wealth;

f. Rank projects with different repayment speed in order to ensure the maximization of shareholders' wealth;

g. Rank projects with different sizes in order to ensure the maximization of shareholders' wealth;

h. Rank projects with different costs of capital in order to ensure the maximization of shareholders' wealth;

i. Simple to compute, easy to interpret and flexible in its use in order to be accepted in practice.

Do managers' preferred methods meet all these conditions? Before answering this question, we quickly describe the general principles of calculation of the main investment criteria.

Consider an investment with a project lifetime of T periods, generating a series of negative (costs) and positive (revenues) cash flows, noted as C_t and R_t respectively with $t = 0, \dots, T$. In order to compare his project to what he could earn on the market for an investment of the same risk class, the manager discounts the cash flows of the project at the cost of capital[5], noted as r. The NPV method, developed by Fisher (1907, 1930) in his pioneering works, consists in determining the difference between the present value of revenues and the present value of costs. The NPV decision rule is to accept all projects with a positive NPV. Between two mutually exclusive projects, the manager must choose the one with the largest NPV. The IRR, developed implicitly by Böhm-Bawerk (1891) and then supported by Keynes (1936), is the discount rate for which the NPV of the project is zero. With the IRR decision rule, the manager has to accept all projects that have an IRR higher than the cost of capital. Between two mutually exclusive projects, the manager must choose the one with the greatest

[5] The question of the determination of the cost of capital is not addressed in this paper. This problem is widely treated in corporate finance textbooks (see, for example, Brealey et al. 2004, Ross et al. 2005).

IRR. Finally, the PI is the ratio between the present value of cash inflows and the present value of cash outflows. The decision rule of the PI is to accept all projects with a PI greater than 1. Between two mutually exclusive projects, the manager must choose the one with the biggest PI.

$$NPV = \sum_{t=0}^{T} R_t \left(1+r\right)^{-t} - \sum_{t=0}^{T} C_t \left(1+r\right)^{-t}$$

$$\sum_{t=0}^{T} R_t \left(1+IRR\right)^{-t} - \sum_{t=0}^{T} C_t \left(1+IRR\right)^{-t} = 0$$

$$PI = \frac{\sum_{t=0}^{T} R_t \left(1+r\right)^{-t}}{\sum_{t=0}^{T} C_t \left(1+r\right)^{-t}}$$

Consider again the conditions of efficiency described above. Financial literature teaches us that the NPV and the PI combine all these qualities except for the condition g: the NPV ensures the maximization of shareholders' wealth when the capital is unlimited and the PI when the capital is rationed. Although both methods fulfil all the required conditions, managers have not adopted the PI in practice. Instead, they widely use the IRR knowing, or not, that this criterion does not meet all the conditions mentioned above.

Nevertheless, it has been long established that the IRR presents two major drawbacks. First, the IRR technique can lead to multiple solutions (Samuelson 1937, Lorie and Savage 1955, Hirshleifer 1958) and thus to a non-unique decision rule (violation of the condition a). Indeed, according to the polynomial theory and Descartes' rule of signs, a stream of n cash flows with m changes of sign for the cash flows (cases of non-normal projects) can have up to m positive IRR. Thus, the IRR decision rule may vary depending on the cash flows pattern. Following these first works, different methods have been suggested to identify in a simple manner situations leading to multiple IRR (see among others Teichroew et al. 1965, Mao 1966, Jean 1968, De Faro 1978). Unfortunately, the scope of these methods is limited from a practical point of view because these methods don't lead to a unique decision rule. Moreover, they don't solve the second problem with the IRR: the IRR technique may not rank mutually exclusive projects in the same order as the NPV does (Alchian 1955) and so may not be consistent with the objective of shareholders' wealth maximization (violation of the conditions e, f, g and h). Such situations occur with certain values of the discount rate when projects have important differences in terms of size, lifetime, capital repayment speed or when projects require different costs of capital[6]. This problem of ranking can be easily proved by depicting in the same figure the investment profiles, i.e. the functions $NPV = f(r)$, of mutually exclusive projects having such differences.

To overcome these drawbacks, other works have attempted to improve the computing technique of the IRR in order to obtain a single rate of return for each project. Solomon (1956) suggests to modify the implicit reinvestment rate assumption of cash flows by an explicit assumption consistent with the NPV method. Thus, instead of reinvesting the cash flows at the IRR, Solomon suggests to reinvest explicitly the cash flows at the cost of capital. In the case of mutually exclusive projects, Solomon proposes to compound non-initial cash flows at the cost of capital and to obtain a terminal value at a time equal to the last period of the longer lived project. The rate of return of a given project is the rate that equalizes the present value of the found terminal value to the initial cash flow. Although this method gets to a single solution, it does not maximize owners' wealth in the case of non-normal projects. Following the works of Solomon, some variants of the IRR based on a modification of the reinvestment rate assumption have been put forward (see among others: Lin 1976, Beaves 1988, Mc Daniel et al. 1988, Bernhard 1989). According to the authors, these various improvements of the IRR are grouped under the following designations: overall rate of return, average rate of return, marginal return to capital or modified internal rate of return. In this paper, we choose the commonly used designation of "modified internal rate of return".

The general calculation principle of these rates of return is the same. The MIRR is the discount rate that equalizes the terminal value of a project (TV) to its investment base (IB): $IB = TV/\left(1 + MIRR\right)^T$. The main differences between these methods have to do with the treatment of positive and negative cash flows, i.e. the manner in which investment base and

[6]Each difference is more or less included in each project and so produces a more or less important effect.

the terminal value of the project are calculated. On the one hand, positive and negative cash flows of a given year can be aggregated into a single annual net cash flow or treated separately. On the other hand, each net cash flow can be (1) allocated to the investment base, (2) allocated to the terminal value, (3) supposed to fund (or to be funded by) other intermediate cash flows. Finally, the discount rate used to calculate the investment base may differ from the cost of capital and correspond to the borrowing rate[7].

In this paper, we choose not to go into these details and consider the MIRR formula generally used by corporate finance textbooks, managers and spreadsheets. Following this formula, the MIRR is the rate for which the present value of cash outflows is equal to the present value of the compounded future value of cash inflows (terminal value), assuming that the cash inflows are reinvested at the cost of capital, i.e.:

$$\sum_{t=0}^{T} C_t \left(1+r\right)^{-t} = \frac{\sum_{t=0}^{T} R_t \left(1+r\right)^{T-t}}{\left(1+MIRR\right)^{T}}$$

$$MIRR = \left[\frac{\sum_{t=0}^{T} R_t \left(1+r\right)^{T-t}}{\sum_{t=0}^{T} C_t \left(1+r\right)^{-t}}\right]^{1/T} - 1$$

The decision rule of the MIRR is to accept all projects that have a MIRR higher than the cost of capital. Between two mutually exclusive projects, the manager must choose the one with the largest MIRR.

The MIRR is an improvement of the IRR which satisfies the managers' preference for expressing the merits of a project with a percentage, and which solves most of the IRR drawbacks (multiple IRR and most ranking problems[8]). Because of this more realistic reinvestment rate assumption, the MIRR is a better criterion of a project's true profitability

[7]For more details on these different modes of calculus, refer for example to Mc Daniel et al. (1988) and Shull (1994). Concerning these methods, the works of Bernhard (1989) show that the approaches of Lin (1976) and Beaves (1988) lead to unequal rates but to an identical projects ranking.

[8]When projects lifetime differs, it is important to calculate the MIRR on an equal number of periods; otherwise the MIRR comparison does not ensure the maximization of owners' wealth. From a technical point of view, we have to add zero cash flows to the shortest projects until their length are equal to the longest project (Mc Daniel et al. 1988).

than the IRR. However, this criterion is not consistent with the maximization of shareholders' wealth when projects require different costs of capital (violation of the condition h). To our knowledge, this drawback is curiously not treated in the literature and was only highlighted by Fabozzi and Peterson (2003). Nevertheless, due to the determination of the WACC from the CAPM and its adjustment to the risk of the project, the costs of capital used for mutually exclusive projects are likely to be different. In other words, the MIRR is not a criterion on which practitioners can rely on to compare projects of different companies, divisions or even risk classes. In what follows, we propose a method that solves this problem and fulfils all the conditions of efficiency mentioned above.

Our intention is not to develop a method which ranks projects of unequal size in the same order that NPV does. This issue does not present much interest in practice, because the NPV already provides an efficient evaluation in monetary terms that is easy to calculate and to interpret. Although complex methods are still developed, this issue was resolved a long time ago by Fisher and its incremental approach that works with all investment criteria. The principle of this method is to create a fictitious project by calculating for each period the difference between projects A and B, and then to compute the IRR (or even the MIRR) of this fictitious project. If the fictitious project is accepted, i.e. if the incremental IRR (or even MIRR) is higher than the cost of capital, so project A is preferred to project B. Thus, to rank a group of k projects with this method, we have to compare $k\,(k-1)\,/2$ pairs of projects, which can quickly become tedious. Our goal is rather to develop a technique that under capital rationing is as efficient as NPV under unlimited capital. So we aim at providing managers the capital budgeting technique complementary to the NPV.

3 The real rate of return method

The MIRR and NPV analysis of competing projects shows that projects with superior MIRR can frequently have lower NPV than projects with inferior MIRR. Such situations occur when projects are of unequal size or when the required costs of capital for these projects are different. To illustrate this latter case, for which we pay a special attention in this paper, we have to imagine two projects (A and B) with the same cash flows pattern but implemented in different countries and/or by different companies. In this manner, we introduce a risk disparity between projects, and so projects differ only by their required cost of capital. If projects A and B are normal, the graphical representations of NPV and MIRR depending on cost of capital ($NPV = f(r)$ and $MIRR = g(r)$) show that NPV is a strictly decreasing function and MIRR a strictly increasing function of the cost of capital. If project A is more risky than project B, we have: $r_A > r_B$, $NPV_A < NPV_B$ and $MIRR_A > MIRR_B$. Therefore, ranking such projects according to their MIRR does not maximize owners' wealth.

To solve this problem of ranking, we need to compare the MIRR of competing projects with respect to their cost of capital. In a trivial sense, we can say that we have to give for each project a measure of the rate of return above the cost of capital. For that, we use the Fisher equation (1907, 1930) establishing the relationship between the nominal rate of return (r_n) on a financial asset, inflation rate (π) and the real rate of return (r_r) on this financial asset: $(1 + r_n) = (1 + r_r)(1 + \pi)$. Considering that the cost of capital is the expected inflation rate of the project, i.e. the expected growth in percentage of the expected average value of investments belonging to the same risk class as the project, and that the MIRR is the nominal rate of return on the project, the Fisher equation allows us to calculate the real rate of return (RRR) of the project[9]:

$$(1 + MIRR) = (1 + RRR)(1 + r) \text{ or } RRR = \frac{(1 + MIRR)}{(1 + r)} - 1$$

What is the economic interpretation of the RRR? Due to its theoretical construct, the RRR corresponds to the rate of return adjusted for the minimum acceptable expected rate of

[9]Notice that the RRR can be easily approximate by subtracting the cost of capital to the MIRR: $RRR \approx MIRR - r$. These approximation works best when both the MIRR and the cost of capital are small. For higher values of the MIRR and the cost of capital, the approximation error (e) becomes large: $e = RRR \times r$ with RRR and r expressed as decimals.

return on a project given its risk. In other words, the RRR is the rate of return at which the firm's value increases in real terms. The RRR differs from the IRR and the MIRR because the latter do not directly express how shareholders' wealth will really increase compared to an investment of the same risk class. The original aspect of the RRR is that it provides managers the same signal as the PI and the NPV: a positive RRR means directly (without comparison) that the return is more than enough to compensate for the opportunity cost of capital. Above all, the RRR is a better investment criterion than other yield-based methods (IRR and its variants) because it gives the same projects ranking as the one obtained with the PI.

Proposition 1 *The RRR is an investment criterion consistent with the objective of shareholders' wealth maximization under capital rationing.*

Proposition 2 *The RRR is the annual rate of return at which one euro has to be compounded for the T periods of the project lifetime in order to get the PI.*

Proof 1 *The formula defining the MIRR is:*

$$(1+MIRR)^T = \frac{\sum_{t=0}^{T} R_t (1+r)^{T-t}}{\sum_{t=0}^{T} C_t (1+r)^{-t}} \tag{1}$$

We know that the present value (PV) of a sum and its terminal value (TV) verify this equality: $TV = PV(1+r)^T$. So, if $PV = \sum_{t=0}^{T} R_t (1+r)^{-t}$ and $TV = \sum_{t=0}^{T} R_t (1+r)^{T-t}$, the following expression is verified:

$$\sum_{t=0}^{T} R_t (1+r)^{T-t} = (1+r)^T \sum_{t=0}^{T} R_t (1+r)^{-t} \tag{2}$$

Substituting (2) *into the equation* (1) *and rearranging, we obtain:*

$$(1+MIRR)^T = \frac{(1+r)^T \sum_{t=0}^{T} R_t (1+r)^{-t}}{\sum_{t=0}^{T} C_t (1+r)^{-t}}$$

$$\frac{(1+MIRR)^T}{(1+r)^T} = \frac{\sum_{t=0}^{T} R_t (1+r)^{-t}}{\sum_{t=0}^{T} C_t (1+r)^{-t}}$$

$$PI = \left(\frac{1+MIRR}{1+r}\right)^T$$

$$PI = \left[1+\left(\frac{1+MIRR}{1+r}-1\right)\right]^T$$

$$IP = (1+RRR)^T \ \ or\ RRR = PI^{1/T}-1$$

So, the RRR can be calculated either from the MIRR or from the PI. For any project with $\sum_{t=0}^{T} C_t (1+r)^{-t} > 0$, the PI values are included in the interval $[0; +\infty[$. On this interval, the RRR is a strictly increasing monotone function which tends to -1 when the PI tends to 0 and to $+\infty$ when the PI tends to $+\infty$ ($RRR \in [-1; +\infty[$). In consequence, for any couple of mutually exclusive projects A and B, if $PI_A > PI_B$ then $h(PI_A) > h(PI_B)$. Thus, the rankings of projects from the RRR and from the PI are similar. As a result, the ranking of equal size projects from the RRR or the NPV is the same; the scale problem not being able to be resolved with a logical rate of return (except with the incremental approach of Fisher).

Proposition 3 *The RRR is consistent with the NPV criterion for projects of equal size.*

Proof 2 *Consider two projects A and B of equal size, i.e.* $\sum_{t=0}^{T} C_{At} (1+r)^{-t} = \sum_{t=0}^{T} C_{Bt} (1+r)^{-t} > 0$*, we then can write:*

$$NPV_A > NPV_B$$

$$\sum_{t=0}^{T} R_{At} (1+r)^{-t} - \sum_{t=0}^{T} C_{At} (1+r)^{-t} > \sum_{t=0}^{T} R_{Bt} (1+r)^{-t} - \sum_{t=0}^{T} C_{Bt} (1+r)^{-t}$$

$$\frac{\sum_{t=0}^{T} R_{At} (1+r)^{-t}}{\sum_{t=0}^{T} C_{At} (1+r)^{-t}} - \frac{\sum_{t=0}^{T} C_{At} (1+r)^{-t}}{\sum_{t=0}^{T} C_{At} (1+r)^{-t}} > \frac{\sum_{t=0}^{T} R_{Bt} (1+r)^{-t}}{\sum_{t=0}^{T} C_{At} (1+r)^{-t}} - \frac{\sum_{t=0}^{T} C_{Bt} (1+r)^{-t}}{\sum_{t=0}^{T} C_{At} (1+r)^{-t}}$$

Substituting $\sum_{t=0}^{T} C_{At} (1+r)^{-t}$ *for* $\sum_{t=0}^{T} C_{Bt} (1+r)^{-t}$ *on the right of this inequality and* *rearranging, we obtain:*

$$\frac{\sum_{t=0}^{T} R_{At} (1+r)^{-t}}{\sum_{t=0}^{T} C_{At} (1+r)^{-t}} > \frac{\sum_{t=0}^{T} R_{Bt} (1+r)^{-t}}{\sum_{t=0}^{T} C_{Bt} (1+r)^{-t}}$$

$$(1 + RRR_A)^T > (1 + RRR_B)^T$$

$$RRR_A > RRR_B$$

So, if $\sum_{t=0}^{T} C_{At} (1+r)^{-t} = \sum_{t=0}^{T} C_{Bt} (1+r)^{-t} > 0$*, then* $RRR_A > RRR_B$ *means* $NPV_A > NPV_B$*.*

Finally, we sum up the modes of calculus and the decision rule of the RRR in the following table:

Modes of calculus	**Situations**	**Decision rule**
$RRR = \sqrt[T]{PI} - 1$	$RRR > 0$	Accept
$RRR = \frac{1+MIRR}{1+r} - 1$	$RRR = 0$	Indifferent
	$RRR < 0$	Reject
	$RRR_A > RRR_B$	Project A preferred to project B

4 A comparison of the MIRR and the RRR bias

In this section, we show another interesting feature of the RRR over the MIRR when cash flows are uncertain. Following Anderson and Barber (1994) who have previously found that the MIRR overstates a project's expected random variable MIRR when cash flows are uncertain, we demonstrate that the RRR bias is inferior to the MIRR bias.

Consider a project with a life of T periods ($t = 0, \ldots, T$), a cost of capital r and an initial investment I_0 followed by a stream of random cash flows $\tilde{F}_1, \ldots, \tilde{F}_T$. The MIRR and the

RRR are the rate of return defined in terms of the expected cash flows:

$$MIRR = \left[\frac{1}{I_0}\sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{T-t}\right]^{1/T} - 1 \tag{3}$$

$$RRR = \frac{1}{(1+r)}\left[\frac{1}{I_0}\sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{T-t}\right]^{1/T} - 1 \tag{4}$$

The random variable MIRR and the random variable RRR on a project may be written as follows:

$$\widetilde{MIRR} = \left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{T-t}\right]^{1/T} - 1 \tag{5}$$

$$\widetilde{RRR} = \frac{1}{(1+r)}\left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{T-t}\right]^{1/T} - 1 \tag{6}$$

Or after simplifying:

$$\widetilde{MIRR} = (1+r)\left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{-t}\right]^{1/T} - 1 \tag{7}$$

$$\widetilde{RRR} = \left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{-t}\right]^{1/T} - 1 \tag{8}$$

When cash flows are certain, it is obvious that equations (3) and (5) are equivalent and that equations (4) and (6) are also equivalent.

When cash flows are uncertain, we demonstrate that the MIRR exceeds the expected random variable MIRR and that the RRR exceeds the expected random variable RRR. Consider f and g two functions of $\tilde{F}_1, \ldots, \tilde{F}_t$ with:

$$\widetilde{MIRR} = f\left(\tilde{F}_1, \ldots, \tilde{F}_t\right) \text{ and } MIRR = f\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_t\right)\right]$$

$$\widetilde{RRR} = g\left(\tilde{F}_1, \ldots, \tilde{F}_T\right) \text{ and } RRR = g\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

Because f and g are concave functions (root functions), then by Jensen's inequality the following inequalities are verified:

$$E\left[f\left(\tilde{F}_1, \ldots, \tilde{F}_T\right)\right] < f\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

$$E\left[g\left(\tilde{F}_1, \ldots, \tilde{F}_T\right)\right] < g\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

Thus, we conclude that:

$$E\left(\widetilde{MIRR}\right) < \ MIRR \text{ and } E\left(\widetilde{RRR}\right) < \ RRR$$

In order to determine if the RRR is a better capital budgeting method than the MIRR when cash flows are uncertain, we have to approximate and compare the MIRR and the RRR bias. For that, we express equations (7) and (8) in terms of the present value of the stream of random cash flows at the cost of capital, $\tilde{P} = \sum_{t=1}^{T} \tilde{F}_t (1+r)^{-t}$, and we use Taylor series expansions to approximate about the expected value of $\tilde{P}$, denoted $\bar{P}$:

$$\widetilde{MIRR} = u\left(\tilde{P}\right) = (1+r)\left(\frac{\tilde{P}}{I_0}\right)^{1/T} - 1$$

$$\widetilde{RRR} = v\left(\tilde{P}\right) = \left(\frac{\tilde{P}}{I_0}\right)^{1/T} - 1$$

Thus we have:

$$\widetilde{MIRR} = u\left(\bar{P}\right) + u'\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right) + \frac{1}{2}u''\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^2 + \ldots + \frac{1}{n!}u^n\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^n + R_n\left(\tilde{P}\right) \tag{9}$$

$$\widetilde{RRR} = v\left(\bar{P}\right) + v'\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right) + \frac{1}{2}v''\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^2 + \ldots + \frac{1}{n!}v^n\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^n + S_n\left(\tilde{P}\right) \tag{10}$$

where $u^n\left(\bar{P}\right)$ and $v^n\left(\bar{P}\right)$ are the n^{th} derivatives of $u(.)$ and $v(.)$ evaluated at $\bar{P}$; and $R_n\left(\tilde{P}\right)$ and $S_n\left(\tilde{P}\right)$ are the remainder terms.

As the expected value of $\tilde{P}$ is $\bar{P} = \sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{-t}$, then $u\left(\bar{P}\right) = MIRR$ and $v\left(\bar{P}\right) = RRR$. Consider that σ_n^n is the n^{th} moment of $\tilde{P}$ about $\bar{P}$ ($\sigma_2^2 = E\left(\tilde{P} - \bar{P}\right)^2$ and $\sigma_3^3 = E\left(\tilde{P} - \bar{P}\right)^3$ are respectively the variance and skewness of $\tilde{P}$), the expectations of equations (9) and (10) can be expressed as follows:

$$E\left(\widetilde{MIRR}\right) = MIRR + \frac{1}{2}u''\left(\bar{P}\right)\sigma_2^2 + \frac{1}{6}u'''\left(\bar{P}\right)\sigma_3^3 + \ldots + \frac{1}{n!}u^n\left(\bar{P}\right)\sigma_n^n + R_n\left(\tilde{P}\right)$$

$$E\left(\widetilde{RRR}\right) = RRR + \frac{1}{2}v''\left(\bar{P}\right)\sigma_2^2 + \frac{1}{6}v'''\left(\bar{P}\right)\sigma_3^3 + \ldots + \frac{1}{n!}v^n\left(\bar{P}\right)\sigma_n^n + S_n\left(\tilde{P}\right)$$

By differentiating $u(.)$ and $v(.)$, we find that $u^n\left(\bar{P}\right) = (1+r)\,v^n\left(\bar{P}\right)$. So, the MIRR and RRR bias may be written as follows:

$$MIRR - E\left(\widetilde{MIRR}\right) = -\frac{1}{2}(1+r)\,v''\left(\bar{P}\right)\sigma_2^2 - \frac{1}{6}(1+r)\,v'''\left(\bar{P}\right)\sigma_3^3 - \ldots - \frac{1}{n!}(1+r)\,v^n\left(\bar{P}\right)\sigma_n^n - R_n\left(\tilde{P}\right) \tag{11}$$

$$RRR - E\left(\widetilde{RRR}\right) = -\frac{1}{2}v''\left(\bar{P}\right)\sigma_2^2 - \frac{1}{6}v'''\left(\bar{P}\right)\sigma_3^3 - \ldots - \frac{1}{n!}v^n\left(\bar{P}\right)\sigma_n^n - S_n\left(\tilde{P}\right) \tag{12}$$

Factoring equation (11) by $(1+r)$ and neglecting the remainder terms, we find:

$$MIRR - E\left(\widetilde{MIRR}\right) = (1+r)\left[RRR - E\left(\widetilde{RRR}\right)\right]$$

Thus we can conclude that the MIRR bias is $(1+r)$ times higher than the RRR bias.

To give a simple approximation of the expected random variable MIRR and expected random variable RRR, we neglect for the sake of simplicity the third and higher terms in equations (11) and (12). As Anderson and Barber (1994) have previously stated for the MIRR case, these approximations are accurate if the variance of the distributions are finite and if higher moments are not extreme. Differentiating twice $v(.)$, we have:

$$v''\left(\bar{V}\right) = -\frac{(T-1)}{T^2}\left(\frac{1}{I_0}\right)^{1/T}\left(\bar{V}\right)^{1/T-2} \tag{13}$$

Substituting (13) into equations (11) and (12) and rearranging terms, we obtain these following approximations of the MIRR and RRR bias:

$$MIRR - E\left(\widetilde{MIRR}\right) = (1+r)\frac{(T-1)}{2T^2}CV^2 PI^{1/T}$$

$$RRR - E\left(\widetilde{RRR}\right) = \frac{(T-1)}{2T^2}CV^2 PI^{1/T}$$

where $CV = \frac{\sigma_2}{V}$ is the coefficient of variation (scaled standard deviation) of the present value of future cash flows and $PI = \frac{\bar{P}}{I_0}$ is the profitability index.

When cash flows are uncertain, we show that the RRR is a better method than the MIRR because the RRR bias is inferior to the MIRR bias. This property may be important for post-audit because when comparing the project's results with expectations, auditors do not necessarily take into account this bias and rely solely on rates of return defined in terms of the expected cash flows. As post-audit is one of the most important elements in a good capital budgeting system, using a less biased investment criterion is preferable to design better capital budgeting programs.

5 Conclusion

Recent capital budgeting surveys indicate that practitioners prefer yield-based methods (IRR and its variants) although they can lead in certain cases to decision errors. In this paper, we develop a new capital budgeting method, named the RRR, which is consistent with the objective of shareholders' wealth maximization. Developed to solve the MIRR failure to maximize the value of the firm when costs of capital differ between projects, the RRR is an improvement of the MIRR and also a variant of the PI. Our starting point is that both the IRR and the MIRR overstate in a certain manner the true profitability of projects compared to the PI. Using the Fisher equation, we define the RRR as the MIRR adjusted for the cost of capital. Calculable in two different ways, the RRR is the annual rate of return at which one euro has to be compounded for the T periods of the project lifetime in order to get the PI. In this way, we show that the RRR solves as the PI the investment decision under capital rationing. When cash flows are uncertain, we also find that the RRR is less biased than the MIRR which is an important feature especially for post-audit. Thus, the RRR is the complementary yield-based investment criterion to the NPV allowing managers to tackle consistently the capital budgeting problem.

In addition to maximize the value of the firm, the RRR combines many advantages and so has a good chance to be accepted by managers in practice. Indeed, it is an easy to calculate and interpret criterion that satisfies managers' needs, habits and way of thinking. It is also a flexible criterion in its use because managers can adapt it to many practices and situations. Firstly, the RRR can be adapted to different assumptions for the calculus of the investment bases and the terminal values of projects: different treatments of positive and negative cash flows, different borrowing and reinvestment rates, etc. Secondly, the RRR is relevant to evaluate mutually exclusive projects that are regularly replaced. Thirdly, this criterion provides managers a good perception of projects risk allowing for the implementation of traditional risk studies as sensitivity analysis and Monte Carlo simulations.

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