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Funding Universal Service Obligations with an Essential Facility: Charges vs. Taxes and Subsidies*

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Abstract

This paper compares three schemes for funding Universal Service Obligations in network industries with an essential facility: an uplift to the network access charge, the establishment of a Universal Service (US) fund financed through a lump-sum tax and a US fund financed through a unit tax. The comparison is made under a duopoly structure with a potential entrant and an incumbent, which owns the essential facility and is responsible for universal service. The incumbent is also constrained to offer the same price on all markets. Using a social welfare criteria, we show that the US fund financed with a lump sum tax dominates the other two schemes, while the US fund with unit tax is equivalent to the access charge uplift.

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1 Introduction

In recent years, many countries have implemented regulatory reforms into their public utility sectors, such as telecommunications, electricity and postal services. The general orientation of these reforms is to move away from franchised monopolies toward more open markets by removing some or all existing barriers of entry.

With free entry and exit in markets, however, some non-profitable markets is at risk of losing service. For equity as well as efficiency purposes,[1] governments often include in the regulatory reforms some programs insuring that all consumers will keep access to the public utility services, i.e. insuring Universal Service (US). A common way of doing this is to mandate one firm to serve some non-profitable segments of the market and to provide a financial compensation for this so-called Universal Service Obligation (USO). The USO mandate can impose either one or both of the following constraints (Chone et al. [3]): "The ubiquity constraint [which] states that all consumers should be connected to a network, whatever their location. The nondiscrimination constraint [which] states that the same tariff should be proposed to all those consumers, whatever their location or their connection cost".

In this paper, we analyze three schemes for funding US under both ubiquity and non-discrimination constraints. In the first scheme, funding is obtained through an uplift to the access charge that suppliers must pay for using the incumbent's essential facility. This access charge then does more than compensate for the cost of network usage; it implicitly contains a supplementary tax aimed at subsidizing non-profitable markets. In the second scheme, a fund is established in order to finance activities on loss-making markets, while the access charge is dedicated to network usage compensation on profitable markets. This fund is raised through a

[1]Efficiency considerations that can lead to government intervention include the presence of positive externalities, such as the network externalities prevalent in the communications industry. In this paper, however, we focus on cases where service to a community is warranted (total utility is greater than total cost) although providing service is not profitable for the firms because of their incapacity of practicing perfect price discrimination. This possibility has been outlined, for instance, by Kahn [5], p.132.

lump-sum tax on profitable markets and is used to finance a unit and/or a lump-sum subsidy for loss-making markets. The third scheme is identical to the second except that the fund is raised through a unit tax. For the three schemes, we assume that USOs are allocated to the incumbent monopoly that existed before the regulatory reform.[2] We also assume that this incumbent owns an essential facility (e.g. the network) that any entrant must access to deliver service to consumers.

We show, on the one hand, that the US fund with unit tax is equivalent to the access charge uplift in the sense that it leads to the same market equilibrium. On the other hand, we prove that welfare is generally the highest under the US fund with a lump-sum tax because this fund scheme uses two instruments, an access charge and a unit subsidy, to reach two different goals: *(i)* recover the network costs and *(ii)* counter the reduction of output that USO provokes by confering some market power to the incumbent. This market power comes from the fact that the incumbent is given a monopoly franchise on a market that is guaranteed to avoid losses by regulation. Although this monopoly power is partly checked by the fact that the incumbent must offer the same price on all markets, an uplift on top of a network break-even access charge would nevertheless exacerbate the downward distortion on output.

This result in fact combines propositions made by Armstrong [1] and Hoernig and Valetti [4]. Focusing on productive efficiency, Armstrong [1] concludes that "retail instruments (perhaps in the form of a carefully designed universal service fund) should be used to combat retail-level distortions such as mandated tariffs that involve cross-subsidies. Wholesale instruments should then be used to combat potential productive inefficiencies".[3] However, this focus on productive efficiency leads him to assume that the price of the incumbent's service is mandated by a regulatory agency and is determined *outside* the model. As we assume that the entrant and incumbent retail costs are identical, we preclude here any possibility of productive inefficiency; we rather focus on allocative efficiency and determine

[2] This is often the case in practice. Note, however, that USOs could be allocated in a number of ways, including auctions. See for instance Anton et al. [1].

[3] p. 301.

endogenously prices that USO funding schemes entail. Our wholesale instrument (access charge) is then used for cost recovery on profitable markets, i.e. markets that would be served without USO, while our "retail instrument" (unit subsidy) is used to counter the fact, observed by Hoernig and Valetti [4] that "a uniform pricing restriction creates linkages between markets...This makes operators less agressive in those markets, leading to higher equilibrium prices and deadweight loss".

Our model is similar to those of Mirabel and Poudou [6] and Chone et al.[3]. A crucial assumption of their papers, however, is that firms are able to practice perfect price discrimination.[4] "Non-profitability" of a market then means that aggregate consumers' and producers' surplus on such a market is negative. As a result, government intervention to impose universal service must be justified by equity or efficiency considerations which are *outside* the models: within the models' logic, a market is not served if and only if it is not socially optimal to have service. Here, we rather assume that it is socially optimal to serve each market, but that some markets are not profitable because firms are unable to extract enough surplus from transactions with their consumers. Using the same demand functions as in Mirabel and Poudou[6] and in Chone et al.[3], this translates in the assumption that firms are constrained to relate on linear pricing.

Compared to non-linear pricing, the requirement of uniform prices will reduce the initial advantage of the incumbent. As a result, under our benchmark case of free competition without USO, firms obtain zero profit.[5] Then, the primary effect of the imposition of USO is to legally institute cross-subsidization that could not be sustained under competition. This amounts to a "reduction of contestability" that will benefit both firms, as they will be able to gain positive profits[6]. USO

[4]Because consumers have homogeneous preferences in these models, perfect price discrimination is attained by two-part tarriffs. Results of the models, however, depend on the capacity of the firms of practicing perfect price discrimination.

[5]In Mirabel and Poudou [6], the benchmark case of competition without USO allows the incumbent to earn a positive profit which amounts to the difference between the total surplus that the incumbent can extract and the total surplus that the potential entrant can extract.

[6]Under the US fund, however, the entrant's profit may eventually be taxed away through a lump sum tranfer to the fund.

would then be supported by the firms and, unsurprisingly, by consumers not served without USO, the targeted beneficiaries of USO. Consumers that would be served without USO pay for the cross-subsidies and are thus the losers of USO.

Section 2 describes the model. Section 3 presents the equilibrium that would prevail without USO, which represents our benchmark case. Sections 4 and 5 derive equilibria under the access charge uplift, the US fund, respectively. Welfare comparisons of the schemes are done in Section 6. Section 7 offers some concluding remarks.

2 Model

2.1 Cost, Demand and Profit Functions

A network industry supplies an homogeneous good that is not storable. Two types of consumers are served, distinguished by their fixed connection cost to the network. For instance, the fixed connection cost of one particular consumer could depend on geographical location (rural vs. urban regions). We denote this fixed cost by $F(\mu)$, where μ takes either value L or H depending on whether the consumer lives in a low fixed cost area or a high fixed cost area, respectively: $F(L) < F(H)$. Proportions of consumers of types L and H are α_L and α_H, respectively, with $\alpha_L + \alpha_H = 1$. For simplicity, we assume that the marginal cost of producing the good and using the network is zero, so that total cost of supplying the good to a consumer of type μ is $F(\mu)$.

Consumers' preferences are identical and are represented by the demand function $q(\cdot)$. This demand is twice differentiable and is such that marginal revenue is always decreasing with quantity. Social welfare W is then measured as the sum of consumers' and producers' surpluses:

$$W(p_L, p_H) = \int_{p_L} \alpha_L q(p)dp + \int_{p_H} \alpha_H q(p)dp - \bar{F} \tag{1}$$

where $\bar{F} = \alpha_L F(L) + \alpha_H F(H)$

It is assumed that the connecting costs of type-H consumers are so high compared to the revenues that can be obtained from these consumers that no enterprise ever finds profitable to serve this type of consumers without subsidies or other forms of help from government. Letting $\pi(p,\mu)$ be the profit from serving a consumer of type μ at price p, this assumption means that:

$$\pi(p,H) = pq(p) - F(H) < 0,\ \forall p > 0$$

We call market μ the set of consumers of type μ, $\mu = \{L, H\}$. Demand of market μ is then $\alpha_\mu q(\cdot)$. We assume that a single supplier can make a non-negative profit by serving both markets at a uniform price. Let p^0 be the monopoly price on both markets, i.e. let p^0 be such that $p^0 q'(p^0) + q(p^0) = 0$, this profitable market assumption implies that

$$\alpha_L \pi\left(p^0, L\right) + \alpha_H \pi\left(p^0, H\right) = p^0 q\left(p^0\right) - \bar{F} \geq 0 \tag{2}$$

Following Mirabel and Poudou [6], we assume a duopoly where firms are indexed by $i \in \{I, E\}$. One firm $i = I$ is an incumbent that owns the network and has a legal obligation to serve type-H consumers. This obligation is compensated by a governmental scheme to help finance the market-H activities. By law, the incumbent must provide third party access to the other firm $i = E$, a (potential) entrant. Access is provided to the regulated price a per unit.[7]

There is then accounting separation of the incumbent's production activities and the supply of network facilities. Incumbent's profit from network access to a consumer of type μ is:

$$\pi_n(p,\mu) = aq(p) - F(\mu)$$

and this profit is obtained independently of the fact that the good is produced and sold by the incumbent or the entrant. In the case the incumbent takes charge of production and distribution to the consumers, the production/distribution profit is given by:

$$\pi_d(p,\mu) = (p-a)q(p)$$

[7] This price is called the access charge.

i.e.the “distribution division” of the firm pays the “network division” for network access. Of course, this transfer within the same firm does not impact on the firm’s global profit. As a result, the incumbent’s profit π_I is independent of the access charge whenever it serves the market:

$$\pi_I(p,\mu) = \pi_d(p,\mu) + \pi_n(p,\mu) = pq - F(\mu)$$

The entrant’s profit function from serving a consumer is:

$$\pi_E(p;a) = (p-a)q(p)$$

We consider a three-stage game. In the first stage, government chooses the relevant parameters for the universal service funding, i.e the level of the access charge and/or various subsidies and taxes, depending on the funding scheme. The choice is made in order to insure that the incumbent’s network activities break even. In the second stage, the incumbent chooses the price of output, acting as a Stackelberg leader[8] vis-à-vis the entrant. In the third stage, the entrant sets its price.

2.2 Universal Service, Funding Schemes and Payoff Functions

Universal Service Obligations are meant to provide a minimum quality to all potential customers. It is thus a set of constraints – or regulation – imposed on the service supplier. Of course, these constraints can vary a lot from one jurisdiction to the other. We focus on two widely used obligations: geographic ubiquity and nondiscrimination. In the context of our model, ubiquity means that the incumbent is mandated to serve market H, while non-discrimination means that the incumbent has to post the same price for service on both markets.

[8] In the theoretical literature on USO, it is standard to assume such an industrial structure, where an incumbent has a leadership with regard to another firm, the entrant (see for example [1]). This leadership could stem from an historical position, a competitive advantage or a commitment decision

Constraint 1 (Non-discrimination): *The incumbent posts the same price* p_I *on both markets* $\mu \in \{L, H\}$.

Note that this constraint does not imply that the price is the same on both markets as the entrant could undercut the incumbent on market L. Since, in our model, the demand function is the same on both markets, the ubiquity constraint boils down to:

Constraint 2 (Ubiquity): $q(p_H) > 0$

Together, constraints 1 and 2 imply that market L is also served: consumers of market L are as willing to consume at price p_H than those of market H and they could even enjoy a lower price from the entrant.

If markets are deregulated, the US providers must be compensated for the cost they incur; otherwise, cream skiming will occur on profitable markets and the US provider will be left with loss-making markets. For instance, the current EU regulatory framework for telecommunications allow for two funding options. "The first is to levy supplementary charges on top of regular interconnection charges, and the second is to create a US fund. The Commission has clearly stated that it prefers the second option, whereby the US cost is paid out of the State budget, by eligible participants, or by end users through a tax"(Hoernig and Valetti[4], p. 8). Accordingly, we consider the following funding schemes that include the supplementary charges option (access charge uplift) and two versions of the US fund that are differentiated by the way the fund is raised.[9]

Access charge uplift. Under this scheme, the access charge plays the dual role of making an entrant pay for the service it uses (market L access) and financing universal service. In other words, the access charge is set by government in order

[9] However, we exclude from the analysis the possibility of raising fund directly from the State budget and impose self-financing from the industry (which corresponds formally to the government budget constraint below). The reason is that the desirability of State financing depends solely on the "shadow cost" of public fund, which is necessarily exogenous in a partial equilibrium model as ours.

to cover overall network costs. Receipts from access charges accrue directly to the incumbent. This was the temporary mechanism applied in France for electricity between February 1999 and February 2000.

US funded with a lump-sum profit tax. The access charge is exclusively dedicated to compensate the incumbent for network usage on the competitive market, but a lump sum tax is levied on market L profit in order to finance a US fund. The lump-sum must be low enough to insure that the market is still served by the entrant or the incumbent. The fund can be used in two ways to incite the incumbent to serve market H: a lump-sum subsidy S and/or a unit subsidy s. Market licences that are differentiated according to profit expectations are examples of lump-sum funding.

US funded with a unit tax. The access charge is again used only to compensate for market L network costs, but the US fund is now raised through a unit tax t levied on both markets.[10] The fund serves again to finance a lump-sum or unit subsidy. For instance, France now charges a unit consumption tax on electricity whose receipts are dedicated for a US fund– the so-called FSPPE[11] In terms of our model, this means that the access charge is used only to compensate for market L network costs, but the US fund is now raised through a unit tax to finance a lump-sum and/or a unit subsidy.

We constrain both funding schemes to be "self-financing" in the sense that total subsidy payment cannot exceed total tax receipts T. We then have the following

[10]We use the accounting convention that a unit tax must be levied on both markets. This convention is in the spirit of deregulation where the network is legally considered as a separate entity from the incumbent, either because of separate ownership or because of regulation imposing separate accounting. Since the network cost is not included in the incumbent's distribution activity, formally, the incumbent does not incur network costs and must pay a tax (and the access charge) for market H operations as it, or the entrant, would for market L.

[11]The use of a consumption tax rather of a production tax is warranted to avoid a competitive disadvantage to domestic producers from international competition on internal markets. If all firms are domestic, as it is implicitly assumed in our model, a consumption tax is strictly equivalent to a production tax and we do not distinguish between them.

government budget constraint:

$$B(s,t,S,T) = s\alpha_H q(p_H(\cdot)) + S - tq\left(p_L(\cdot)\right) - T \geq 0 \tag{3}$$

where $p_\mu(a,s,t,S,T)$ are equilibrium prices for given tax and subsidy parameters.

The following table, where $\pi_L(a,s,t,S,T)$ denotes the equilibrium profit on market L and $Q \equiv \alpha_L q(p_L) + \alpha_H q(p_H)$, presents schematically these schemes.

	Charges and taxes	**Form of US payment**	**Constraints**
Access charge uplift	both markets: a	included in a	$a = \frac{F}{Q}$
US/Lump Sum Tax	both markets: a market L: T	S, s	$a = \frac{F(L)}{\alpha_L q}$ $\pi_L \geq 0,\ B \geq 0$
US/Unit tax	both markets: a, t	S, s	$a = \frac{F(L)}{\alpha_L q}$ $\pi_L \geq 0,\ B \geq 0$

Different schemes are thus different combinations of lump-sum tranfers and unit taxes or subsidies. Sometimes, they can be indistinguishable from the point of view of one of the firm. For instance, from the point of view of the entrant, a given unit tax is equivalent to an access charge of the same amount. However, the access charge is not equivalent to the unit tax for the government because their uses are not constrained in the same way: the access charge is constrained of covering network costs, while the unit tax is constrained by market L overall profit. Moreover, the revenue from the access charge accrue directly to the incumbent, while the revenue of the unit tax is placed in a fund to be redistributed afterwards to the incumbent in the form of a unit or lump-sum subsidy. These differences can potentially impact on incentives given for service on market H.

Differences in the funding schemes essentially modify the firms' payoff functions of the firms. We first define a general payoff functions for both firms that account of all possible instruments used by the government (a, t, T, s and S). Payoff functions for a given funding scheme will be a particular case of these general payoff functions where some instruments are set to zero. The strategic variables are each firm's price. Since goods from both firms are perfectly homogeneous from the consumers' point of view, the firm that announces the lowest price can serve the entire market if it wishes.[12] The entrant will then capture all the market if it announces a price lower than the incumbent. Letting $\tau \equiv a + t$, its payoff function is then:[13]

$$\Pi_E(p_E; p_I, \tau, T) = \begin{cases} (p_E - \tau)\alpha_L q(p_E) - T & \text{if } p_E \leq p_I \\ 0 & \text{if } p_E > p_I \end{cases} \tag{4}$$

The entrant chooses p_E in order to maximize of Π_E. This leads to a reaction function $R^E(p_I; \tau, T)$ that is taken into account by the incumbent and the government. The incumbent receives any subsidy s or S for serving market H as well as the access charges. In counterpart, it has to pay the unit tax t for its output sold on either market L or H, and lump-sum tax T if it serves market L. Market L is served by the incumbent whenever it posts a price that is lower than the entrant's. The incumbent's profit function is thus:

$$\Pi_I(p_I, R^E(\cdot); a, t, T, s, S) =$$
$$\begin{cases} (p_I + (s-t))\alpha_H q(p_I) + a\alpha_L q(R^E(p_I; a)) - \bar{F} + S & \text{if } p_I \geq R^E(\cdot) \\ (p_I + (s-t))\alpha_H q(p_I) + (p_I - t)\alpha_L q(p_I) - \bar{F} + S - T & \text{if } p_I < R^E(\cdot) \end{cases} \tag{5}$$

The incumbent then chooses price p_I in order to maximize Π_I. This leads to a reaction function $R^I(a, t, T, s, S)$ that is taken into account by the government.[14]

In many cases, the firm optimal choice of strategy will involve the determination of a monopoly price with an appropriately chosen implicit marginal cost. In other

[12] Since both firms will always face constant marginal costs, if it is profitable to serve part of the market at a given price, it is more profitable to serve the entire market. Thus the firm with the lowest price will serve the entire market.

[13] We consider that the entrant wins the market if it exactly matches the incumbent's price. If we rather consider that, under a price tie, the market is served by the incumbent or is shared between firms, quantities, prices and welfare stay identical.

[14] Note that the incumbent is forced to serve market H by the government, so that we do not include a participation constraint $\Pi_I \geq 0$. In fact, the funding schemes are constrained to insure that network costs are covered so that the participation is implicitly taken into account when the government sets parameters (a, t, T, s, S).

words, optimal choice of strategy of either firm will be the result of one or many problems of the form:

$$\max_{p}(p-c)q(p) \tag{6}$$

where c will be a function of the scheme parameters (a, t, T, s, S). The optimal choice will then have the form:

$$p^*(c) \equiv \{p^* \,|q(p^*) + (p^* - c)\, q'(p^*) = 0\}$$

or equivalently

$$p^*(c) \equiv \left\{ p^* \left| \frac{p^* - c}{p^*} = \frac{1}{\eta(p^*)} \right. \right\} \tag{7}$$

where $\eta(p) \equiv -q'p/q$ is the price elasticity of demand. This is of course the standard "inverse elasticity rule" for monopoly pricing. The second order condition is:

$$2q'(p^*) + (p^* - c)q''(p^*) < 0$$

As c will be implicitly defined by the various schemes parameters, we will want to compare the price p^* for different values of c. Comparative statics lead to:

$$p^{*\prime}(c) = \frac{dp^*}{dc} = \frac{q'}{2q' + (p^* - c)q''} > 0$$

Note that monopoly price p^0 was the result of solving problem (6) for the particular case where marginal cost is zero: $p^0 = p^*(0)$.

2.3 Entrant's Best Reply

From the entrant's payoff function (4), we see that if $p_E > p_I$, the profit is nil, so this reply gives us zero as a lower bound to the profit in the case that the entrant chooses $p_E \leq p_I$. So the entrant's choice of price can be written as the following constrained problem:

$$\begin{aligned} &\max_{p_E}\ (p_E - \tau)\alpha_L q(p_E) - T \\ &s.t. \\ &\qquad p_I - p_E \geq 0 \\ &\qquad (p_E - \tau)\alpha_L q(p_E) - T \geq 0 \end{aligned} \tag{8}$$

Let $\lambda \geq 0$ be the Lagrange multiplier associated with constraint $p_I - p_E \geq 0$. We solve the problem by ignoring first the participation constaint and check afterwards whether it is met or not. The FOC (sufficient here) are

$$\alpha_L \left[q(p_E) + (p_E - \tau)\, q'(p_E) \right] - \lambda = 0 \tag{9}$$

$$\lambda \left(p_I - p_E \right) = 0 \tag{10}$$

Two cases must then be considered.

Case 1 $\lambda = 0$ *and* $p_E \leq p_I$

Then, from (9), the optimal solution p_E^* is characterized as follows:

$$p_E^* = \left\{ p_E \left| \frac{p_E - \tau}{p_E} = \frac{1}{\eta} \right. \right\} = p^*(\tau)$$

This is the case where the incumbent's price is so high that it allows the entrant to get the monopoly price considering that the entrant's marginal cost is τ. If the lump sum tax T does not swamp the monopoly gross profit $\alpha_L(p_E^* - \tau)q(p_E^*)$, then the entrant effectively chooses p_E^*. Otherwise, there is formally no solution to problem (8), which means in our context that the entrant excludes itself from the market by choosing any price $p_E \in (p_I, \infty)$.

Case 2 $\lambda > 0$ *and* $p_E = p_I$

Then, from (9), we have:

$$p_I q'(p_I) + q(p_I) = \tau q'(p_I) + \frac{\lambda}{\alpha_L} > \tau q'(p_I)$$

which implies that $p_I < p^*(\tau)$. This is the case where p_I is below the entrant's monopoly price $p^*(\tau)$, so that the best reply to p_I is simply to match the price, considering that the lump-sum tax T then leaves a non-negative profit. If such a price leads to a negative profit, the entrant rather chooses any price in the interval (p_I, ∞).

The entrant's reaction function to the incumbent's price p_I is thus:

$$R^E(p_I;\tau,T) = \begin{cases} p^*(\tau) & \text{if} \quad p_I \geq p^*(\tau) > \tau \text{ and } T \leq \alpha_L(p^*(\tau)-\tau)q(p^*(\tau)) \\ p_I & \text{if} \quad p^*(\tau) > p_I \geq \tau \text{ and } T \leq \alpha_L(p_I-\tau)q(p_I) \\ p_E \in [p_I,\infty) & \text{otherwise} \end{cases} \tag{11}$$

2.4 Incumbent's Best Reply

Refering to the incumbent's payoff function (5), we see that two cases must be considered.

Case 1 $p_I < R^E(\cdot)$

Given the entrant's reaction function, we have that $p_I < \tau$ or $T > \alpha_L(R^E(\cdot) - \tau)q(R^E(\cdot))$. In other words, the entrant does not match the price because its margin is negative at this price or entry is excluded by a "prohibitive" lump-sum tax. In either case, the incumbent is a monopolist on both markets. It thus solves the following problem:

$$\underset{p_I \in [0,R^E(\cdot))}{\arg\max}\, \Pi_I\left(p_I, p_I; a, t, T, s, S\right)$$

The necessary first order condition then leads to:

$$p_I = \left\{ p \left| \frac{p-t+\alpha_H s}{p} = \frac{1}{\eta} \right. \right\} = p^*\left(t - \alpha_H s\right)$$

As $t - \alpha_H s$ represents the effective marginal cost to the incumbent, this is the usual inverse elasticity rule for a monopolist. Let $p^M(s,t) \equiv p^*\left(t - \alpha_H s\right)$. The incumbent's profits are then[15]

$$\Pi_I = (p^M + (s-t))\alpha_H q(p^M) + (p^M - t)\alpha_L q(p^M) - \bar{F} + (S - T)$$

Price p^M thus represents the monopoly price when competition is made ineffective because of high taxes. This case will thus prevail whenever $p^M < \tau$ or $T > \alpha_L(p^M(s,t) - \tau)q(p^M(s,t))$.

The following result is used in case 2.

[15] We omit the arguments of Π_I and $p^M(\cdot)$ for ease of presentation.

Lemma 1 $p^M(s,t) < p^*(\tau)$

Proof. Since $t - \alpha_H s \leq t < a + t = \tau$, we have $p^M(s,t) = p^*(t - s\alpha_H) < p^*(\tau)$. ■

Case 2 $p_I \geq R^E(p_I; \tau, T)$ *and* $T \leq \alpha_L(R^E(\cdot) - \tau)q(R^E(\cdot))$

Then the entrant serves market L. Let

$$p^D(a,s,t) = \underset{p_I \in [\tau, \infty)}{\arg\max} \Pi_I(p_I, R^E(\cdot); a, t, T, s, S) \tag{12}$$

To find $p^D(a,s,t)$, we first note by contradiction that it is impossible to have $p^D(a,s,t) > p^*(\tau)$. Suppose it were the case. Since $p^*(\tau) > p^M(s,t)$, the incumbent could reduce its price to $p^*(\tau)$ without seeing any reaction from the entrant. This would increase profit on market H, without modifying profit from market L, showing by contradiction that the profit maximizing price cannot exceed $p^*(\tau)$. Then we must have $p^D(a,s,t) \leq p^*(\tau)$. In that case, from (11), the entrant chooses $R^E(p_I, \tau, T) = p_I$. The incumbent then maximizes $\Pi_I(p_I, p_I; a, t, T, s, S)$ under the constraints that $\tau \leq p_I \leq p^*(\tau)$. Assuming for the moment that these constraints are not binding, we have[16]:

$$\left\{ p^D(a,s,t) \left| \frac{p^D(a,s,t) - (t-s) + \frac{\alpha_L}{\alpha_H} a}{p_I^D(a,s,t)} = \frac{1}{\eta} \right. \right\} = p^* \left(t - s - \frac{\alpha_L}{\alpha_H} a \right)$$

We then get the following result [17]

Lemma 2 $p^D(a,s,t) < p^M(s,t)$

Proof. Since $t - s - \frac{\alpha_L}{\alpha_H} a < t - \alpha_H s$, we have $p^*(t - s - \frac{\alpha_L}{\alpha_H} a) < p^*(t - \alpha_H s) = p^M(s,t)$. Since, from Lemma 1, $p^M(s,t) \leq p^*(\tau)$, this means that constraint $p^D(a,s,t) \leq p^*(\tau)$ is never binding. ■

[16] We later check whether it is in fact the case.

[17] Since $p^D(a,s,t) < p^M(s,t) < p^*(\tau)$, the constraint $p_I < p^*(\tau)$ is satisfied and $p^D(a,s,t)$ is then the solution to the incumbent's problem.

However, as $p^*(t-s-\frac{\alpha_L}{\alpha_H}a)$ is decreasing in $s+\frac{\alpha_L}{\alpha_H}a$, nothing warants that constraint $p^*(t-s-\frac{\alpha_L}{\alpha_H}a)\geq\tau$ is met. Hence we get:

$$p^D(a,s,t)=\max\left\{p^*\left(t-s-\frac{\alpha_L}{\alpha_H}a\right),\tau\right\} \tag{13}$$

Summing up the results, we have:

$$R^I(a,t,T,s,S)=\begin{cases} p^D(a,s,t) & \text{if} \quad \tau\leq p^M(s,t) \text{ and } T\leq\alpha_L(p^M(s,t)-\tau)q(p^M(s,t)) \\ p^M(s,t) & \text{if} \quad \tau>p^M(s,t) \text{ or } T>\alpha_L(p^M(s,t)-\tau)q(p^M(s,t)) \end{cases} \tag{14}$$

3 Benchmark Case: No Universal Service Obligation

As a benchmark case, we consider first that the government does not impose universal service obligations. This means first that $t=T=s=S=0$. More importantly, this also means that market H will not be served and that consequently, competition is restricted to market L.

3.1 Second and Third Stages: Firms' Choices

The entrant reaction function is given directly from (11) with $T=0$ and $\tau=a$. Since, without lump-sum tax, the entrant does not face any fixed cost, the only decisive factor for entry or not becomes whether p_I is greater than a or not. We thus have:

$$R^E(p_I;a,0)=\begin{cases} p^*(a) & \text{if} \quad p_I\geq p^*(a)>a \\ p_I & \text{if} \quad p^*(a)>p_I\geq a \\ p_E\in[p_I,\infty) & \text{otherwise} \end{cases} \tag{15}$$

Rigorously, the incumbent profit and reaction functions must in fact be reworked: by construction, market H will not be served so that it is virtually inexistant. But we can use an "as if" argument[18] to derive the incumbent reaction function in that case:

[18] See appendix A for a more rigourous but equivalent derivation of the best reply.

since market H will not be served and no USO are imposed, it is rather equivalent to the limit case where nobody is of type H i.e. it is as if $\alpha_H = 0$. Taking the limit for $\alpha_H \to 0^+$ in (14) and with $t = T = s = S = 0$, leads to

$$R^I(a,0,0,0,0) = \begin{cases} p^D(a,0,0) = a & \text{if} \quad a \leq p^*(0) \\ p^M(0,0) = p^*(0) & \text{if} \quad a > p^*(0) \end{cases}$$

where $p^D(a,0,0)$ was obtained from (13): $p^D(a,0,0) = \max\left\{\lim_{\alpha_H \to 0} p^*\left(-\frac{\alpha_L}{\alpha_H}a\right), a\right\} = a$. Moreover $\alpha_L(p^*(0) - a)q(p^*(0)) \lesseqgtr 0$ if $a \gtreqless p^*(0)$. So the incumbent reaction function writes (recall that $p^0 = p^*(0)$) as:

$$R^I(a,0,0,0,0) = \begin{cases} p^0 & \text{if} \quad a > p^0 \\ a & \text{if} \quad a \leq p^0 \end{cases}$$

3.2 First Stage: Government's Choice

The government is commited to make the incumbent's network activities break even[19]. An access charge higher than p^0 would permit the incumbent to get monopoly profits and, implicitly, the network activities would generate a positive profit since market L has been assumed profitable: $\alpha_L \pi_n(q, L) = \alpha_L(aq(p^0) - F(L)) > \alpha_L(p^0 q(p^0) - F(L)) > \alpha_L(p^0 q(p^0) - F(L) - F(H)) > 0$. The access charge must then be less than p^0. In order to have zero profit for network activities, the government solves the implicit equation:

$$a = \frac{F(L)}{q_L(a)}$$

With such an access charge, the incumbent sets $p_I = a$ and obtains a zero profit for its network activities. The entrant gets the market with zero profit.

In brief, without USO, the profitable market is contestable: the potential entrant can make "hit-and-run" entry and exit as its cost is composed only of the variable cost a. With the access charge equal to the network average cost, the market is

[19] In this paper, informationnal problems are ignored, despite the fact that the incumbent has a strong incentive to misreport the low fixed costs.

then disciplined to zero-profit. Presumably, this is the kind of results envisonned when network industry deregulations are advocated for. However, as the imposition of USO implicitly amounts to restore the cross-subsidies that existed before deregulations, and since cross-subsidies are unsustainable in contestable markets, US necessarily will weaken the market constestability promoted by deregulation.

4 Universal Service Funded by an Access Charge Uplift

We now assume that the government wants to insure US. It forces the incumbent to serve market H and to post the same price for service on both markets. This precludes price discrimination. In return, the government gives the assurance that the access charge will allow the overall network activities to break even and it forbids entry on market H. Network cost recovery implies that the access charge will equal aggregate market average fixed cost.

4.1 Second and Third Stages: Firms' Choices

In the access charge scheme, there are no explicit taxes or subsidies so that $t = T = s = S = 0$. For the entrant, the situation is thus similar to the case of no universal service obligation and the reaction function is given by (15). For the incumbent, we replace the tax and subsidy values into (14). Since there is no lump-sum profit tax, market L is necessarily profitable at monopoly price $p^M(0,0) = p^0$. The incumbent's reaction function then boils down to

$$R_u^I(a) \equiv R^I(a,0,0,0,0) = \begin{cases} p^D(a,0,0) & \text{if} \quad a \leq p^0 \\ p^0 & \text{if} \quad a > p^0 \end{cases} \tag{16}$$

Proposition 1 *At equilibrium under access charge uplift,* $p_L = p_H \equiv p^D(a,0,0)$.

Proof. Since both markets are assumed to be profitable together, this means that monopoly price p^0 is higher than average cost at that price, $\left[\alpha_L F(L) + \alpha_H F(H)\right]/q(p^0)$. We thus have $a < p^0$. From (16), the incumbent then chooses $p_I \equiv p^D(a,0,0) \geq a$. From (11), the entrant follows with $p_E = p_I$. ■

4.2 First Stage: Government's Choice

Let $p^u = p^D(a,0,0)$. Since Government is commited to choose a^u such that the overall network profit will be nil, we have

$$a^u \equiv \frac{\alpha_L F(L) + \alpha_H F(H)}{q(p^u)} \tag{17}$$

This access charge thus includes a cost component, $\alpha_H F(H)$ that is not related to the network segment open to competition. The entrant will then be asked to pay for access an amount greater than the cost of the service they receive, namely the market L network. This is the sense given to "uplift".

Fixing the access charge thus involves solving two equations, (16) with $a \leq p^0$ and (17).[20] Profit of the incumbent is then

$$\Pi_I\left(p_I^u, p_I^u, \frac{\alpha_L F(L) + \alpha_H F(H)}{q\left(p_I^u\right)}, 0,0,0,0\right) = \alpha_H\left[p_I^u q(p_I^u) - \alpha_L F(L) - \alpha_H F(H)\right] \geq 0 \tag{18}$$

while profit of the entrant is

$$\alpha_L\left[p_I^u q(p_I^u) - \alpha_L F(L) - \alpha_H F(H)\right] \geq 0 \tag{19}$$

Both companies share the total market fixed cost in proportion of their market. This comes from the fact that, by setting the access-charge equal to both market average fixed costs, the regulator has created a level-playing field for both firms. Linkages between both markets created by the non-discriminatory price constraint, however, renders the incumbent less agressive on market L; a profit is made by the

[20] We assume here that there exists a positive solution in (p_I, a) for this set of equations.

entrant on this market, showing that the market is not contestable.[21]

We now present a condition that determines whether the industry profit is positive or nil.

Lemma 3 *Let $\bar{p}$ be such that $\bar{p} = \frac{\bar{F}}{q(\bar{p})}$. (i) If $\alpha_H > \eta(\bar{p})$,[22] then $p_I^u > a^u$ and firms' profit is positive; (ii) if $\alpha_H \leq \eta(\bar{p})$, then $p_I^u = a^u = \bar{p}$ and firms' profit is nil.*

Proof. (i) Assume that $\alpha_H > \eta(\bar{p})$ and that the equilibrium solution is $p_I^u = a^u = \bar{p}$. Then, from (5) with $p_I > R^E(\cdot)$ and $s = t = 0$, we can write the incumbent's marginal profit of the incumbent at $\bar{p}$ as :

$$\begin{aligned}
\frac{\partial \Pi_I}{\partial p_I^u} &= \bar{p}\alpha_H q'(\bar{p}) + \alpha_H q(\bar{p}) + \bar{p}\alpha_L q'(\bar{p}) \\
&= \bar{p} q'(\bar{p}) + \alpha_H q(\bar{p}) \\
&> \bar{p} q'(\bar{p}) + \eta(\bar{p}) q(\bar{p}) = 0
\end{aligned}$$

But this means that increasing the price would increase profit, proving that $p_I^u = a^u = \bar{p}$ is not an equilibrium. The equilibrium is then such that $p_I^u \equiv p^D(a^u, 0, 0) > a^u$. With $p_I^u > \frac{\bar{F}}{q(p_I^u)}$, each firm's profit is positive.

(ii) Assume that $\alpha_H \leq \eta(\bar{p})$. Using the same line of reasoning as in (i), we obtain $\frac{\partial \Pi_I}{\partial p_I^u} \leq 0$ at $\bar{p}$. But as the incumbent's price cannot be chosen below $\bar{p} = a^u$, $\bar{p}$ is the equilibrium price. ■

The intuition behind Lemma 3 is as follows. When the incumbent increases its price, revenues are lost from market L access charges receipts because of the decrease of demand on market L. Such a price increase is thus beneficial only if the incumbent operates in the inelastic portion of the demand, so that a price increase brings a revenue increase, and if market H is sufficiently large (and consequently, market

[21] Welfare maximization under both non-discrimination constraint and non-negative profit constraint would lead to average cost pricing : $p_L = p_H = \left\{ \bar{p} | \, \bar{p} = \frac{\alpha_L F(L) + \alpha_H F(H)}{q(\bar{p})} \right\}$. This price is higher than the price prevailing (on market L) without USO (which was $\left\{ p | \, p = \frac{\alpha_L F(L)}{q(p)} \right\}$) but it would leave no economic profit to the firms.

[22] Note that, since the uplift makes the incumbent perceive a "negative" marginal cost, the equilibrium price will in general be situated in the inelastic portion of the demand.

L is relatively small), so that this revenue increase can compensate lost revenue on market L. On contrary, if the percentage of the population which lives in H is lower than the percentage of demand reduction following a price increase, the revenue gain $\alpha_H q$ of selling a given quantity at a higher price will not compensate for the revenue loss associated to the loss of demand for the whole population $\bar{p}q'(\bar{p})$. Then, price will be set at the lowest price possible and markets can be considered contestable. As a particular case, if $\alpha_H = 0$, then we retrieve the no universal service solution found in the preceding section.

We thus see that the access charge uplift can maintain constestable market results provided that markets to be protected by universal service are not too large.

5 Universal Service Fund

We now consider the case where the government establishes a US fund. The fund is raised in order to compensate for USO with a unit subsidy and/or a lump-sum subsidy on market H. The entrant is not eligible to the subsidies, so that market H is monopolistic. The access charge is maintained to insure that network activities on market L, i.e.

$$a = \frac{F(L)}{q(p_L)} \tag{20}$$

The fund is raised either by a lump-sum subsidy or a unit tax.

5.1 Second and Third Stages: Firm's Choices

The scheme we consider is one where $a > 0$, $T \geq 0$, $t \geq 0$, $s \geq 0$ and $S \geq 0$. The reaction function of the entrant is thus given by (11) and the incumbent reaction function is given by (14). We can then characterize the equilibrium in the second stage.

Proposition 2 *At equilibrium under Universal Service Fund,* $p_L = p_H \equiv p^D(a, s, t)$.

Proof. We proceed by contradiction. Suppose that the incumbent can sustain the monopoly price $p^M(s,0)$. Then,

$$\begin{aligned}
0 &\leq \left(p^M(s,t) + s\alpha_H - t\right)\left[q\left(p^M(s,t)\right)\right] - \bar{F} + S - T \\
&\leq \left(p^M(s,t) - t\right)\left[q\left(p^M(s,t)\right)\right] - \bar{F} && \begin{array}{r}(\text{since } s\alpha_H p^M(s,0)q(p^M(s,0)) + S - T \\ = tq\left(p^M(s,t\right) \geq 0)\end{array} \\
&\leq \left(p^M(s,t) - t\right)\left[q\left(p^M(s,t)\right)\right] - F(L) && (\text{since } F(L) < \bar{F}) \\
&\leq \left(p^M(s,t) - a - t\right)\left[q\left(p^M(s,t)\right)\right] && (\text{since } a = \tfrac{F(L)}{q(p^M(s,t))})
\end{aligned}$$

which implies that $p^M(s,t) \geq a + t = \tau$. But this contradicts the fact that to sustain this monopoly price, we must have $\tau > p^M(s,t)$. Then, no equilibrium exists with $\tau > p^M(s,t)$. Government must then choose a, s and t such that $\tau \leq p^M(s,0)$. The incumbent then chooses $p_I = p^D(a,s,t) \geq \tau$ and the entrant follows with $p_E = p_I$.
■

The exact value of the equilibrium price then depends on the rules used to fix the taxes. We consider in turn the lump-sum tax and the unit tax.

5.2 First Stage with a Lump-Sum Tax

When the fund is raised strictly with the lump-sum tax T, we have $t = 0$. Let $p^T \equiv p^D(a,s,0)$ be the equilibrium price, the access charge is then given by:

$$a^T \equiv \frac{F(L)}{q(p^T)} \tag{21}$$

Then two cases are considered by the government. If profits made on market L are high enough, it will raise a fund T sufficient to compensate entirely the incumbent for network costs. If market L profits are too low to reach such a goal, it will raise the highest fund possible, which amounts to market L profit, to compensate as much as possible the incumbent for the network activities.

Assume first that market L profits are sufficient to pay for aggregate network costs and let (s^+, S^+) be the subsidy vector that allows for network cost recovery.

Then, (s^+, S^+) is such that:

$$a^T \alpha_L q(p^T) + (a + s^+)\alpha_H q(p^T) + S^+ = \bar{F} \tag{22}$$

Given rule (21) for the setting of the access charge, we obtain:

$$s^+ \alpha_H q(p^T) + S^+ = \alpha_H (F(H) - F(L))$$

This transfer is funded through a lump sum tax T^+ on market L supplier:

$$T^+ = s^+ \alpha_H q(p^T) + S^+ \tag{23}$$

This scenario is valid whenever the amount of such a lump-sum tax is lower than market L profit i.e. whenever

$$T^+ = \alpha_H (F(H) - F(L)) < \alpha_L (p_L - a) q\left(p_L\right) = \alpha_L p_L q\left(p_L\right) - \alpha_L F(L) \tag{24}$$

Assume second that (24) is not verified, so that government taxes away profit on market L:

$$T^0 = \alpha_L p_L q\left(p_L\right) - \alpha_L F(L) \tag{25}$$

Then the subsidy vector (s^0, S^0) is constrained as follows

$$s^0 \alpha_H q(p_I) + S^0 = \alpha_L (p_L q\left(p_L\right) - F(L))$$

Lemma 4 *Usage of a lump-sum subsidy can never increase the welfare associated to the US fund.*

Proof. Suppose we have an equilibrium with $S > 0$, $T > 0$ and $s \geq 0$. We have then two cases to consider.

(i) The equilibrium is such that there is complete compensation of network cost.

We first note that, when the case of complete compensation of network costs prevail, $p_I^T = p^D(a, s, 0)$ is necessarily greater than a. We thus have $p^D(a^T, s^+, 0) = p^* \left(-s^+ - \frac{\alpha_L}{\alpha_H} a^T\right) > a^T$. Assume that we have an equilibrium with $S^+ > 0$. Then, from (23), a small reduction in S^+ could be accompanied by a small increase in

s^+ for a given T^+. Indeed if $S^+ > 0$ then totally differentiating (26), (20) and $p_I^T = p^D\left(a^T, s^+, 0\right)$, we have

$$\frac{dp_I^T}{dS^+}\cdot\left(1+\eta\left(p_I^T\right)\frac{p^{*\prime}(\cdot)}{p_I^T}\left(s^+ + \frac{\alpha_L}{\alpha_H}a^T\right)\right) = \frac{p^{*\prime}(\cdot)}{\alpha_H q\left(p_I^T\right)}$$

thus $\frac{dp_I^T}{dS^+} > 0$. This change $(dS^+ < 0)$ allows to reduce the price $p^D(a^T, s^+, 0) = p^*\left(-s^+ - \frac{\alpha_L}{\alpha_H}a^T\right)$. As this price is initially higher than marginal cost (assumed to be zero), this would lead to an increase of welfare (defined in 1) : since $p^{*\prime}(\cdot) > 0$ and so for $dS^+ < 0$: $\frac{dW\left(p_I^T\right)}{dS^+} > 0$.

(ii) The equilibrium is such that the entrant makes no profit.

We first note that to have a zero profit with $T^0 > 0$, $p_I^T = p^D(a^T, s^0, 0)$ must necessarily be greater than a. We thus have $p^D(a, s, 0) = p^*(-s - \frac{\alpha_L}{\alpha_H}a) > a$. Assume that we have an equilibrium with $S^0 > 0$. Then, from (25), a small reduction in S^0 could be accompanied by a small increase in s^0 for a given T^+. This change allows to reduce the price $p^D(a^T, s^0, 0) = p^*\left(-s^0 - \frac{\alpha_L}{\alpha_H}a^T\right)$. As this price is initially higher than marginal cost (assumed to be zero), this would lead to an increase of welfare[23].

■

- **Complete compensation of network costs**

  With $S^+ = 0$, we get from (26) that:

$$s^+ = \frac{(F(H) - F(L))}{q(p_I^T)} \tag{26}$$

  and, from Proposition 2, that the price is implicitly defined by:

$$p_I^T = p^D(a^T, s^+, 0) = p^*\left(-s^+ - \frac{\alpha_L}{\alpha_H}a^T\right) = p^*\left(\frac{-\bar{F} + \alpha_H F(L)}{\alpha_H q(p_I^T)}\right)$$

  Profit of the entrant is then computed using equations (23) (20) and (26)

$$\Pi_E = \alpha_L\left(p_I^T - a^T\right)q(p_I^T) - T^+ = \alpha_L\left(p_I^T q(p_I^T) - F(L)\right) - \alpha_H(F(H) - F(L)) \tag{27}$$

[23] We prove in appendix B that

$$\frac{dp_I^T}{dS^0} > 0$$

which was assumed positive for this case. Profit of the incumbent is computed using equations (20) and (26):

$$\Pi_I = (p_I^T + s^+)\alpha_H q(p_I^T) + a^T \alpha_L q(p_I^T) - \bar{F} = \alpha_H \left(p_I^T q(p_I^T) - F(L)\right) \qquad (28)$$

This is also positive since a positive Π_E in (27) implies that $p_I^T q(p_I^T) > F(L)$. Instead of sharing the network cost in proportion of market share, as it was the case under the uplift scheme, the entire network cost, net of the part paid by the incumbent for network access on market H ($\alpha_H F(L)$) is transfered to the entrant in order to fund the unit subsidy of the non-profitable market, where production has to be stimulated.

Since the incremental cost of serving market H is paid by the market L supplier, this case is most likely to occur when α_L is relatively large[24]. With a high α_L and a low α_H, the transfer per unit of market L output $T^+/\alpha_L q(p_T^D) = \alpha_H[F(H) - F(L)]/\alpha_L$ could well be small even though $F(H)$ is high relative to $F(L)$. USO would then not seem to be an important problem. For instance, France supplies high cost electricity services to its overseas population at the same price that it does to its low-cost continental consumers. But the proportion of overseas population is so low that, presumably, USO would not impact significantly on a continental competitive market.

- **Zero-profit on market** L

  With $S^0 = 0$, the unit subsidy is given by:

$$s^0 = \frac{T^0}{\alpha_H q(p_I^T)} = \left(p_I^T - \frac{F(L)}{q\left(p_I^T\right)}\right)\frac{\alpha_L}{\alpha_H} \qquad (29)$$

  The equilibrium price can be characterized more fully by using Proposition 2, access charge equilibrium (20) and subsidy equilibrium (29):

$$p_I^T = p^D(a^T, s^+, 0) = p^*\left(-s^+ - \frac{\alpha_L}{\alpha_H}a^T\right) = p^*\left(-p_I^T\right)$$

[24] Remember that we assumed that there exists a price p^0 such that $p^0 q(p^0) - \alpha_L F(L) - \alpha_H F(H) > 0$. Since the revenue of the entrant will be $\alpha_L p_I^T q(p_I^T)$, we see that complete compensation is possible to the extent that α_L is sufficiently large and p_I^T sufficiently "close" to p^0.

which, from the definition of p^* in (7), implies that

$$p_I^T = \{p \,|\eta(p) = \alpha_H\}$$

- The equilibrium price is thus totally independent of cost. This is because we are in a situation where the whole market profit goes to the incumbent, which acknoledge this before choosing its price.

  Profit of the incumbent is then

$$\begin{aligned} \Pi_I &= (p_I^T + s)\alpha_H q(p_I^T) + a^T \alpha_L q(p_I^T) - \alpha_L F(L) - \alpha_H F(H) \\ &= p_I^T q(p_I^T) - \alpha_L F(L) - \alpha_H F(H) \end{aligned} \quad (30)$$

  which is non-negative because p_I^T is bounded below by $\bar{p}$.[25] Profit of the entrant is nil by assumption.

  It turns out that the lump-sum transfer T^0 is equal to the incumbent's opportunity cost of letting the entrant serve the market at price p_I^T rather than serving itself the market at that price: $T^0 = \alpha_L(p_I^T - a^T)q(p_I^T)$. The overall network access charge that the entrant pays for each unit of output is:[26]

$$a^T + \frac{T}{\alpha_L q\left(p_I^T\right)} = \frac{F(L)}{\alpha_L q\left(p_I^T\right)} + \alpha_L(p_I^T - a^T) \quad (31)$$

  The first term represents the average incremental cost to the incumbent of entry in market L, while the second represents the opportunity cost to the incumbent of this entry at price p_I^T. The sum of the two terms can be assimilated to Baumol and Sidak's ECPR rule, with two caveats. First, in our model, the price p_I^T is set endogenously by the incumbent, while, in Baumol and Sidak's model, the price is exogenously set by a regulator. As a result, Baumol and Sidak cannot evaluate the allocative efficiency of their rule, while we can compare it to other funding schemes. Second, Baumol and Sidak include the overall payment (31) in the access charge, while we split it into an

[25] Conditions under which this profit is positive or nil are derived below.

[26] Revenue minus costs on transport activities are then

$$a^T \alpha_L q(p_L) + T - \alpha_L F(L) - \alpha_H F(H) = T - \alpha_H F(H)$$

which is assumed here to be less than or equal to zero.

access charge and a lump-sum tax, and we also split the incumbent receipt into an access-charge and a unit subsidy. This is also related to the fact that the price is set endogenously in our model: market power means that prices will be set inefficiently high by the incumbent. We thus need two independent instruments, a and s,[27] to reach two different goals, network cost recovery and the reduction of allocative inefficiency, respectively.

Note that hitting the market L profit constraint is likely to happen whenever α_H is significant and/or the difference between $F(H)$ and $F(L)$ is important. For instance, in Canada, 70% of the population is concentrated in a small band of territory (the "Quebec-Windsor corridor"), while 30% of the population is dispersed over a huge territory. Insuring service at the same price over the whole territory, as in the postal services, then leads to significant price distortions and cross-subsidies from urban regions to rural ones.

We now provide a condition under which the industry profit wil be nil.Note that contrary to the access charge uplift scheme, $\alpha_H > \eta(\bar{p})$ will not necessarily lead to positive profit.

Lemma 5 *Let $\bar{p}$ be such that $\bar{p} = \frac{\bar{F}}{q(\bar{p})}$. If $\alpha_H \leq \eta(\bar{p})$, then $p_I^T = \bar{p}$ and firms' profit is nil*

Proof. From Lemma 3, if $\alpha_H \leq \eta(\bar{p})$, $p_I^u = \bar{p} = \max\left(\bar{p}, p^*\left(-\frac{\alpha_L}{\alpha_H}\frac{\bar{F}}{q(p_I^u)}\right)\right)$ where p_I^u would be the incumbent's choice of price were it not constrained to choose a price higher than a^u.

- Consider first the case of complete compensation of network costs when we are under the lump-sum tax funding scheme. Then $p_I^T = p^*\left(-\frac{(F(H)-F(L))}{q(p_I^T)} - \frac{a_L}{\alpha_H}\frac{F(L)}{q(p_I^T)}\right)$. Assume that $p_I^T > \bar{p} \geq p_I^u$. This implies that

$$p_I^T = p^*\left(-\frac{(F(H)-F(L))}{q(p_I^T)} - \frac{\alpha_L}{\alpha_H}\frac{F(L)}{q(p_I^T)}\right) > \bar{p} \geq p^*\left(-\frac{\alpha_L}{\alpha_H}\frac{\bar{F}}{q(p_I^u)}\right)$$

[27] Of course, T is not an independent instrument given a and s.

Then, since p^* is monotone increasing we have

$$\frac{\bar{F}-\alpha_H\bar{F}}{\alpha_H q(p_I^T)} < \frac{\bar{F}-\alpha_H F(L)}{\alpha_H q(p_I^T)} = \frac{(F(H)-F(L))}{q(p_I^T)} + \frac{\alpha_L}{\alpha_H}\frac{F(L)}{q(p_I^T)} < \frac{\alpha_L}{\alpha_H}\frac{\bar{F}}{q(p_I^u)} = \frac{\bar{F}-\alpha_H\bar{F}}{\alpha_H q(p_I^u)}$$

implying that $q(p_I^T) > q(p_I^u)$, in contradiction with the initial assumption that $p_I^T > p_I^u$. We thus have $p_I^T = \bar{p}$. Then both firms' profit is nil.

- Consider second the case of zero profit of the entrant. Since we have just shown that no profit is possible for the incumbent even when we consider initially that the entrant non-negative profit constraint is not binding, no profit is possible when the entrant non-negative profit is binding. In fact, the two cases collapses in this case with $p_I^T = \bar{p}$ as the only feasible solution.

■

Lemma 6 *If* $\alpha_H > \eta(\bar{p})$,[28] *then* $p_I^T > \bar{p}$ *and firms' profit is positive.*

Proposition 3 *The equilibrium price* p_I^T *is characterized as follows:*

$$p_I^T = \begin{cases} \bar{p} & \text{if } \alpha_H \leq \eta(\bar{p}) \\ \{p \,|\eta(p) = \alpha_H\} & \text{if } \alpha_H > \eta(\bar{p}) \text{ and } \alpha_L\left(p_I^T q(p_I^T) - F(L)\right) < \alpha_H(F(H)-F(L)) \\ p^*\left(\frac{-\bar{F}+\alpha_H F(L)}{\alpha_H q(p_I^T)}\right) & \text{if } \alpha_H > \eta(\bar{p}) \text{ and } \alpha_L\left(p_I^T q(p_I^T) - F(L)\right) \geq \alpha_H(F(H)-F(L)) \end{cases}$$

5.3 First Stage with a Unit Tax

5.4 First Stage: Government's choice

The government chooses the access charge so that profit from market L transport is zero:

$$a^t \equiv \frac{F(L)}{q\left(p^t\right)} \tag{32}$$

[28] Note that, since the uplift makes the incumbent perceive a "negative" marginal cost, the equilibrium price will in general be situated in the inelastic portion of the demand.

Total transfers made to the incumbent must cover the loss due to transport activities on market H:[29]

$$s\alpha_H q(p^t) + S = \alpha_H(F(H) - aq(p^t)) \tag{33}$$

This transfer is funded through a per unit tax on both markets output. Then the tax is

$$t = \frac{s\alpha_H q(p^t) + S}{q(p^t)} \tag{34}$$

We assume for the moment that this tax rate leaves a positive profit for market L operations. We will check later that this is in fact the case at equilibrium.

Government has two instruments to make a subsidy for US service: a unit subsidy and a lump-sum subsidy. It is shown in Appendix C that, in fact, the lump-sum subsidy is useless. Intutively, this is because the lump-sum subsidy, while being financed through a distortionary tax, does not allow to incite producers to increase output as can do a unit subsidy and as is wished in a monopoly and duopoly structure.

Lemma 7 *There always exists an equilibrium with $S = 0$.*

Proof. See appendix C. ■

With $S = 0$, we get from (33) and (34) that

$$s = \frac{t}{\alpha_H} = \frac{[F(H) - F(L)]}{q(p_I^t)}$$

Note also that

$$\tau = a^t + t = \frac{\bar{F}}{q(p_I^t)}$$

With this solution, profit of the incumbent is

$$\begin{aligned} &\Pi_I\left(p_I^t, p_I^t; \frac{F(L)}{q\left(p_I^t\right)}, \frac{\alpha_H\left[F(H) - F(L)\right]}{q(p_I^t)}, 0, \frac{F(H) - F(L)}{q(p_I^t)}, 0\right) \\ =\ &\alpha_H\left[p_I^t q(p_I^t) - \alpha_L F(L) - \alpha_H F(H)\right] \geq 0 \end{aligned} \tag{35}$$

[29]Technically, the constraint is $s\alpha_H q(p_I) + S \geq \alpha_H(F(H) - aq(p_I))$, but since the lump-sum subsidy is financed by a distortionary unit tax, government will always want to give the lowest lump-sum subsidy possible. We take this factor into account immediately for ease of presentation.

while profit of the entrant is

$$\Pi_E = \alpha_L(p_I^t - a - t)q(p_I^t) = \alpha_L \left[p_I^t q(p_I^t) - \alpha_L F(L) - \alpha_H F(H)\right] \geq 0 \tag{36}$$

Proposition 4 *Unit tax funding is equivalent to the access charge uplift.*

Proof. We have that $p_I^t \equiv p^D(a^t, s, t) = \max\left(\left\{p \middle| p = \frac{\bar{F}}{q(p)}\right\}, p^*\left(\frac{\alpha_L}{\alpha_H}\frac{\bar{F}}{q}\right)\right) = p^D(a^u, 0, 0)$. Substituting this price into (35) and (18), we see that the incumbent's profit is the same under both schemes. Similarly, substituting this price into (36) and (19) show that the entrant's profit is the same under both schemes. Finally, government balances budget under the unit tax funding scheme, while no money transit to government in the case of an access charge uplift. ■

The equivalence comes from two sources. First, monopoly pricing is unsustainable in both schemes. If monopoly pricing were an equilibrium in both schemes, incentives provided by the schemes would be different as can be seen from (16) and (14): a unit subsidy $s > 0$ makes the monopoly price $p^M(s, 0)$ lower than the price p^0 that would prevail under the access charge uplift. Second, given that duopoly pricing prevails, the government budget constraint insures that the incentives that can be given through the US fund are identical to those given under the access charge uplift. To see this, consider the government budget constraint (3). As transfers to market H must be financed by market L, we must have, under unit tax funding with $S = 0$, that $s\alpha_H = t$. The total subsidy per unit of market H output is thus :

$$\sigma^t \equiv s - t + \frac{\alpha_L}{\alpha_H}a^t = \alpha_L s + \frac{\alpha_L}{\alpha_H}a^t \tag{37}$$

Since the access charge a^t is meant to finance market L network while the unit subsidy s is meant to finance the incremental fixed cost of serving market H consumers, we have $s = [F(H) - F(L)]/q$ and $a^t = F(L)/q$. Then the total unit subsidy received for market H output is:

$$\begin{aligned}\sigma^t = \alpha_L \frac{F(H) - F(L)}{q} + \frac{\alpha_L}{\alpha_H}\frac{F(L)}{q} &= \frac{\alpha_L}{\alpha_H}\frac{\alpha_H (F(H) - F(L)) + F(L)}{q} \\ &= \frac{\alpha_L}{\alpha_H}\frac{\bar{F}}{q} = \frac{\alpha_L}{\alpha_H}a^u\end{aligned}$$

The incumbent thus receives the same per unit subsidy whether it is under the unit tax scheme or under the access charge uplift. Similarly, for the entrant

$$\tau^t = a^t + t = \frac{F(L)}{q} + \frac{\alpha_H \left[F(H) - F(L)\right]}{q} = \frac{\bar{F}}{q} = a^u$$

i.e. the entrant faces the same marginal cost whether it is under the unit tax regime or under the access charge uplift.

6 Welfare Comparisons

In the following propositions, we compare the various fund systems we have analyzed in sections 5 and 5.3. Since we have just stated in proposition 4, that unit tax funding is equivalent to an access charge uplift, those comparisons are limited to both lump-sum tax and access charge uplift schemes.

First we give the following useful intermediate result.

Lemma 8 *Let $\bar{p}$ be such that $\bar{p} = \frac{\bar{F}}{q(\bar{p})}$. If $\alpha_H > \eta(\bar{p})$, then $p_I^T < p_I^u$, i.e price is lower under lump-sum funding than under the access charge uplift.*

Proof. When $\alpha_H > \eta(\bar{p})$,we have obtained that $p_I^u = p^* \left(\frac{\alpha_L}{\alpha_H} \frac{\bar{F}}{q(p_I^u)} \right)$. For p_I^T, we must consider two cases:

- *Entrant's profit is nil under the US fund*

  Then $p_I^T = p^* \left(-\frac{\alpha_L}{\alpha_H} p_I^T \right)$. Assume that $p_I^T \geq p_I^u$. Then $p^* \left(-\frac{\alpha_L}{\alpha_H} p_I^T \right) \geq p^* \left(-\frac{\alpha_L}{\alpha_H} \frac{\bar{F}}{q(p_I^u)} \right)$ and, since p^* is monotone increasing, this implies that $p_I^T \leq \frac{\bar{F}}{q(p_I^u)}$. We thus have that $p_I^u \leq p_I^T \leq \frac{\bar{F}}{q(p_I^u)}$. But this is contradiction with Lemma 3 which states that industry profit is positive under the access charge uplift scheme when $\alpha_H > \eta(\bar{p})$. As a result, $p_I^T < p_I^u$.

- *Entrant's profit is positive under the US fund*

  Then $p_I^T = p^* \left(-\frac{(F(H)-F(L))}{q(p_I^T)} - \frac{\alpha_L}{\alpha_H} \frac{F(L)}{q(p_I^T)} \right)$. Assume that $p_I^T \geq p_I^u$. Then, since p^* is monotone increasing, we have

  $$\frac{\bar{F} - \alpha_H \bar{F}}{\alpha_H q(p_I^T)} < \frac{\bar{F} - \alpha_H F(L)}{\alpha_H q(p_I^T)} = \frac{(F(H) - F(L))}{q(p_I^T)} + \frac{\alpha_L}{\alpha_H} \frac{F(L)}{q(p_I^T)} < \frac{\alpha_L}{\alpha_H} \frac{\bar{F}}{q(p_I^u)} = \frac{\bar{F} - \alpha_H \bar{F}}{\alpha_H q(p_I^u)}$$

implying that $q(p_I^T) > q(p_I^u)$, in contradiction with the initial assumption that $p_I^T \geq p_I^u$. We must then have $p_I^T < p_I^u$ whenever the entrant's profit is positive.

For both cases, we thus obtain that $p_I^T < p_I^u$. ■

We can now state that prices are never higher under lump-sum funding (with a unit subsidy) than under the access charge uplift.

Proposition 5 *The equilibrium price under a US fund raised through a lump-sum tax does not exceed the equilibrium price under the access charge uplift, i.e.* $p_I^T \leq p_I^u$

Proof. This results directly from the combination of Lemma 5 and Lemma 8. ■

Proposition 6 *Social welfare under lump-sum funding is greater or equal to social welfare under the access charge uplift*

Proof. Since from (1), we have $\frac{dW}{dp_I} = -q\left(p_I\right) < 0$, directly applying 5, leads to the result. ■

Proposition 3 is rather intuitive. Welfare is maximized whenever $p_H = p_L = MC = 0$, where p_H and p_L are prices on markets H and L, respectively, and MC stands for marginal cost. Moreover, with zero marginal cost, maximization of welfare corresponds to maximization of consumers' surplus. Since consumers' surplus is decreasing in price, the lower the price, the higher is social welfare. Let p_μ^u and p_μ^T represent equilibrium prices on market μ under the access charge uplift and lump sum funding. As

$$p_I^T = p_H^T = p_L^T \leq p_H^u = p_L^u = p_I^u$$

welfare is higher under the US fund.

The superiority of lump-sum funding comes from the fact that it reduces the distortionary effect of the access charge. With lump-sum funding, the access charge is lower ($\frac{F(L)}{q}$ compared to $\frac{\bar{F}}{q}$), so that competition from the entrant is more effective, which means that the incumbent's monopoly power from linked markets is made lower. In other words, the part of network costs that is financed through lump-sum funding does not impact on the entrant's behavior and allows to finance a unit subsidy that incites the incumbent to increase the output. Lump-sum funding thus allows to use two instruments: the access charge and a unit subsidy, to meet two objectives, network financing and increased output. The access charge uplift uses only one instrument to meet both objectives. Note also that the fact that lump-sum funding works better than unit tax funding is because the lump-sum funding helps to finance a "genuine" net subsidy: the unit tax only permits to create a subsidy which is the exact equivalent to the uplift of the access charge scheme.

Corollary 1 *If the revenue function $R(p) \equiv pq(p)$ is strictly concave, $\forall p$, industry profit under the access charge uplift is greater or equal to industry profit under lump-sum funding.*

Proof. Let $p^0 = p^*(0)$ be the profit maximizing price for a monopoly. >From the facts that $p_I^T \leq p_I^u < p^0$ and that the profit function is strictly concave[30], we then have that industry profit is higher under the access charge. ■

Because both funding schemes only redistribute profit from one firm to the other, industry profit is the lowest the farther a funding scheme brings the price from p^0. By using better incentives to increase the production, i.e. by making better use of potential competition from the entrant, lump-sum funding has more success in increasing output than does the access charge uplift. As a result, the gap between the equilibrium price and monopoly price is made higher, and profit is lower.

Note also that, in cases where they make a positive profit, both firms prefer the imposition of USO over free competition (no USO)[31]: USO confers monopoly power to the incumbent on a newly profitable market and, with non-discrimination constraint, lessens competition to an otherwise contestable market. In all cases,

[30] This comes from the fact that the revenue function is assumed concave and that the cost function is linear.

[31] In the case that the introduction of a US fund leaves the entrant with no profit, the entrant is indifferent between the fund and no USO. Otherwise, both firms strictly prefer USO.

consumers of the market L, who pay for USO, are losers of its imposition, and consumers of initially non-profitable markets, who are not served without USO, are beneficiaries of USO. As $p_I^T \leq p_I^u$, both market consumers prefer the (lump-sum/unit subisdy) fund over the access charge.

7 Conclusion

Introduction of USO funding schemes reduces the contestability of markets to a point where the incumbent accomodates entry in the profitable market in order to relax the non-discrimination constraint and manage to get a profit on the high cost market.

Welfare can be higher under the US fund than under the uplift (or the US fund with unit tax) because the unit subsidy incites output expansion, and thereby counters the output restriction that the non-discrimination constraint provokes by lessening market constestability on profitable markets. This output expansion, however, reduces industry profits. The incumbent could still prefer the US fund because of the receipts it gets from it; however, the entrant always prefer the access charge uplift.

Low-cost consumers are losers of USO service as they finance them. However, they prefer the US fund over the access charge uplift because the price is lower with unit subsidies. The same reason leads the high-cost consumers to also prefer US fund scenario.

Two extensions should readily be studied. First, as firms can foresee what are the consequences of their actions on the government's choice, one could consider that the government, in fact, plays last. Since government uses rules to set the uplift and the subsidy, one can suspect that the firms can take into account these rules and then manage to set monopoly prices on both markets. This conjecture, however, has to be shown formally. If confirmed, it would send a warning for the long run market power consequences of USOs.

Second, entrant's marginal cost of distribution should be assumed different from the incumbent's and unknown to the incumbent and government. This would allow to consider problems of productive efficiency such as (i) the possibility that entry occurs inefficiently (i.e. entry while the entrant's cost is higher than the incumbent's) or *(ii)* the possibility that entry is blocked inefficiently (i.e. no entry while the entrant's cost is lower than the incumbent's). Although problems of productive efficiency have been studied extensively in the access pricing literature, to our knowledge, there exists no model that looks at US funding while considering both allocative and productive efficiency.

Appendices

A. No USO: incumbent's reaction

The incumbent maximizes its profit given the entrant's reaction function R^E and the access charge a. Its payoff function is then

$$\hat{\Pi}_I(p_I, R^E(p_I; a, 0), a) = \begin{cases} \alpha_L(aq\left(p_E\right) - F\left(L\right)) & \text{if} \quad p_I \geq R^E(p_I; a, 0) \\ \alpha_L\left(p_I q\left(p_I\right) - F\left(L\right)\right) & \text{if} \quad p_I < R^E(p_I; a, 0) \end{cases}$$

We then have two cases to consider:

1. if $p_I < R^E(p_I; a, 0)$, given the entrant's reaction function, it must be the case that $p_I \leq a$ because if the contrary holds ($p_I > a$), it leads to the contradiction $p_I > p_I$. So maximizing $\Pi_I(p_I, p_E, a)$ w.r.t. $p_I \leq a$ when $p_E \in [p_I, \infty]$, is equivalent to solve the following simple constrained problem

$$\begin{cases} \max\limits_{p_I} \alpha_L\left(p_I q\left(p_I\right) - F\left(L\right)\right) \\ a - p_I \geq 0 \end{cases}$$

Directly, its solution is given by

$$p_I = \begin{cases} \left\{p^0 \middle| \, p^0 q'\left(p^0\right) + q\left(p^0\right) = 0\right\} & \text{if } p^0 < a \\ a & \text{if } p^0 \geq a \end{cases}$$

Respectively the incumbent's profits are

$$\begin{aligned}\hat{\Pi}_I^{1a} &= \alpha_L\left(p^0 q\left(p^0\right) - F\left(L\right)\right) \text{ if } p^0 < a \\ \hat{\Pi}_I^{1b} &= \alpha_L\left(aq\left(a\right) - F\left(L\right)\right) \quad \text{if } p^0 \geq a\end{aligned}$$

2. If $p_I \geq R^E\left(p_I; a\right)$ then given the entrant's reaction function, we have two sub-cases to consider:
   (a) $p^*\left(a\right) > p_I = R^E\left(p_I; a\right) \geq a$, the incumbent's profit is $\Pi_I^{2a}\left(p_I\right) = \alpha_L\left(aq\left(p_I\right)\right) - F\left(L\right))$
   (b) $p_I \geq R^E\left(p_I; a\right) = p^*\left(a\right) > a$, the incumbent's profit is $\Pi_I^{2b} = \alpha_L\left(aq(p_E^*\left(a\right)\right) - F\left(L\right))$
   Since for all $p_I < p^*\left(a\right)$, $q\left(p_I\right) > q(p^*\left(a\right))$ then $\Pi_I^{2a}\left(p_I\right) > \Pi_I^{2b}$ and $a = \arg\max_{p_I \geq a} \Pi_I^{2a}\left(p_I\right)$. So the solution is $p_I = a$.

Finally by summing up the results, we see that:

- if $p^0 \geq a$ then $\Pi_I^{1b} = \Pi_I^{2a}\left(a\right)$ and the optimal incumbent's reply is $p_I = a$
- if $p^0 < a$ then $\Pi_I^{1a} > \Pi_I^{2a}\left(a\right)$ and the optimal incumbent's reply is $p_I = p^0$

The incumbent reaction function thus writes (as in the text):

$$R_{nus}^I\left(a\right) = \begin{cases} p^0 & \text{if } \; a > p^0 \\ a & \text{if } \; a \leq p^0 \end{cases}$$

B. Formal proof of lemma 4

If $S^0 > 0$ then totally differentiating (25), (20) and $p_I^T = p^D(a^T, s^0, 0)$ leads to

$$\frac{ds^0}{dS^0} = -\frac{1}{\alpha_H q\left(p_I^T\right)} + \frac{dp_I^T}{dS^0}\left[\eta\left(p_I^T\right)\frac{s^0}{p_I^T} + \frac{\alpha_L}{\alpha_H}\left(1 - \eta\left(p_I^T\right)\right)\right]$$

so

$$\frac{dp_I^T}{dS^0} \cdot \left[1 + \eta\left(p_I^T\right)\frac{p^{*\prime}\left(\cdot\right)}{p_I^T}\left(s^0 + \frac{\alpha_L}{\alpha_H}\left(a^0 + p_I^T\frac{1 - \eta\left(p_I^T\right)}{\eta\left(p_I^T\right)}\right)\right)\right] = \frac{p^{*\prime}\left(\cdot\right)}{\alpha_H q\left(p_I^T\right)} \qquad \text{(B.1)}$$

But from the definition of $p^D(a^T, s^0, 0)$ in (13), in turns out that

$$p_I^T\frac{1 - \eta\left(p_I^T\right)}{\eta\left(p_I^T\right)} = s^0 + \frac{\alpha_L}{\alpha_H}a^0$$

substituing in (B.1), this yields

$$\frac{dp_I^T}{dS^0}\cdot\left[1+\eta\left(p_I^T\right)\frac{p^{*\prime}(\cdot)}{\alpha_H p_I^T}\left(s^0+\frac{\alpha_L}{\alpha_H}a^0\right)\right]=\frac{p^{*\prime}(\cdot)}{\alpha_H q\left(p_I^T\right)}$$

which leads to

$$\frac{dp_I^T}{dS^0}>0$$

C. Formal proof of lemma 7

(i) Assume that we have an equilibrium with $S>0$ and $p_I^t=p^D(a,s,t)=\tau$. From (32) and (33), we can easily form

$$p_I^t=\tau=\frac{\bar{F}}{q(\tau)}$$

Differentiating it and doting variables to denote the derivate with respect to S (that is $\dot{a}=\frac{da}{dS}$, $\dot{t}=\frac{da}{dS}$, $\dot{s}=\frac{ds}{dS}$, $\dot{\tau}=\frac{d\tau}{dS}$), proves that this price is invariant to the subsidy manipulations

$$\dot{\tau}=\dot{\tau}\eta\left(\tau\right)\Rightarrow\dot{\tau}=0$$

Moreover similarly we have

$$\dot{t}=\eta\left(\tau\right)\frac{\dot{\tau}}{\tau}t=0\text{ and }\dot{a}=\eta\left(\tau\right)\frac{\dot{\tau}}{\tau}a=0$$

Thus differentiating (34), we see that

$$\dot{s}=-\frac{1}{\alpha_H q(\tau)}$$

Since $\dot{\tau}=0$, manipulating S is not harmful from a social point of view, so $S=0$ is also optimal.

(ii) Assume that we have an equilibrium with $S>0$ and $p_I^t=p^D(a,s,t)>\tau$, differentiating (32), (34), (33) and $p_I^t=p^*\left(t-s-\frac{\alpha_L}{\alpha_H}a\right)$, and doting variables to denote the derivate with respect to S (that is $\dot{a}=\frac{da}{dS}$, $\dot{t}=\frac{da}{dS}$, $\dot{s}=\frac{ds}{dS}$ and $\dot{p}_I^t=\frac{dp_I}{dS}$)

$$\begin{aligned}\dot{a}&=\eta\left(p_I^t\right)\frac{\dot{p}_I^t}{p_I^t}a\\ \dot{t}&=\eta\left(p_I^t\right)\frac{\dot{p}_I^t}{p_I^t}t\\ \dot{t}q(p_I^t)+tq'(p_I^t)\dot{p}_I^t&=\dot{s}\alpha_H q(p_I^t)+s\alpha_H q'(p_I^t)\dot{p}_I^t+1\\ p_I^t&=p^{*\prime}\left(\cdot\right)\left[\dot{t}-\dot{s}-\frac{\alpha_L}{\alpha_H}\dot{a}\right]\end{aligned}$$

Isolating $\dot{s}$ from the second and substituing $\dot{a}$ and $\dot{t}$ gives

$$\dot{s} = s\eta(p_I^t)\frac{\dot{p}_I^t}{p_I^t} - \frac{1}{\alpha_H q(p_I^t)}$$

Then

$$\begin{aligned} \dot{p}_I^t &= p^{*\prime}(\cdot)\left[\eta\left(p_I^t\right)\frac{\dot{p}_I^t}{p_I^t}t - s\frac{\eta(p_I^t)\dot{p}_I^t}{p_I^t} + \frac{1}{\alpha_H q(p_I^t)} - \frac{\alpha_L}{\alpha_H}\eta\left(p_I^t\right)\frac{\dot{p}_I^t}{p_I^t}a\right] \\ &= \eta\left(p_I^t\right)\frac{\dot{p}_I^t}{p_I^t}p^{*\prime}(\cdot)\left(t - s - \frac{\alpha_L}{\alpha_H}a\right) + \frac{p^{*\prime}(\cdot)}{\alpha_H q(p_I^t)} \\ \Rightarrow \; & \dot{p}_I^t\left(1 - \frac{\eta\left(p_I^t\right)}{p_I^t}p^{*\prime}(\cdot)\left(t - s - \frac{\alpha_L}{\alpha_H}a\right)\right) = \frac{p^{*\prime}(\cdot)}{\alpha_H q(p_I^t)} \end{aligned}$$

Using the defintion (13) of $p^D(\cdot)$ this implies

$$\dot{p}_I^t\left(1 - p^{*\prime}(\cdot)\left(\eta\left(p_I^t\right) - 1\right)\right) = \frac{p^{*\prime}(\cdot)}{\alpha_H q(p_I^t)}$$

which proves that $\dot{p}_I^t > 0$ because at the equilibrium $\forall S \geq 0,$ using (32), (34) and (33) we see thta $t - s - \frac{\alpha_L}{\alpha_H}a^t = -\left(\alpha_L s + \frac{S}{q(p_I^t)} + \frac{\alpha_L}{\alpha_H}a^t\right) < 0$ so $\eta\left(p_I^t\right)$ is necessarily lower than 1. To finish the proof, we have just to remember that the welfare is decreasing w.r.t the uniform price p_I^t.

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